JHA, xli (2010)

SOLAR ANOMALY AND PLANETARY DISPLAYS IN THE ANTIKYTHERA MECHANISM*

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Recent scholarship has greatly improved our understanding of the structure and function of the Antikythera mechanism.[1] One unresolved issue is whether the mechanism included a display of the variability (or anomaly) in the angular motion of the Sun. In Price's original reconstruction, the motion of the Sun was treated only in the mean. A single pointer indicated the day of the year in the Egyptian calendar and the position of the uniformly-moving mean Sun in the zodiac. Of course, the actual position of the Sun can depart from that of the mean Sun by up to 2° at some times of year (up to 2°23′ in Hipparchos's solar theory). Wright seems to have been the first to propose that the mechanism treated the solar anomaly.[2] And since the recognition that the mechanism includes a representation of the Moon's variable motion,[3] probably based on an epicycle model like that of Hipparchos, a number of similar proposals for the solar anomaly have been made (although no gear work for the Sun survives except a simple set of gears that produced a uniform angular motion). Generally these proposals involve separate pointers for the mean and true Sun. We will show that the mechanism did, indeed, treat the solar anomaly, but in a more economical fashion. A single pointer sufficed to give the date in the Egyptian year (indicative of the mean Sun) and the place of the true Sun in the zodiac.

A second unresolved issue is the nature of the mechanism's planetary displays. The inscription on the front cover appears to include detailed references to the synodic behaviour of Aphrodite (Venus).[4] Some scholars have therefore proposed that the mechanism gave a complete, kinematic display of the motions of all five known planets around the zodiac. Freeth, Jones, Steele and Bitsakis, for example, wrote, "It seems likely that the Antikythera Mechanism also displayed some or even all of the five planets known in ancient times, but there is considerable debate about this".[5] Michael Wright had made a similar proposal and, in a mechanical *tour de force*, built a working model of the Antikythera mechanism that included pointers for the zodiacal positions of the five planets, along with the Sun and Moon, all with co-axial movement.[6] While this proves that such a design is technically possible, it does not, of course, demonstrate that this is what the builder of Antikythera mechanism actually did (as Wright himself freely grants). It is important to remember that no gear

* This paper is partly based on data processed from the archive of experimental investigations by the Antikythera Mechanism Research Project in collaboration with the National Archaeological Museum of Athens (see Freeth *et al*., "Decoding" (ref. 1)).

trains for the planets survive, except perhaps for a single gear (called r1) of 63 teeth that has no assigned role in the most recent reconstruction. We observe also that a construction that might seem natural and desirable to a twentieth- or twenty-first-century designer may not have seemed so to a mechanic of the second century B.C. — and vice versa. We have had several good examples of this in the century-long effort to understand the Antikythera mechanism. Most notably, the recent discovery that the mechanism included a dial for displaying the four-year cycle of Olympic and other games[7] was almost completely unanticipated.[8] We offer a proposal for the planetary display that is more modest and that ties together the surviving evidence. In particular, we suggest the mechanism may have displayed planetary phases (i.e., key events in the synodic cycle) rather than a kinematic representation of deferent and epicycle theory.

1. THE SOLAR ANOMALY

Representing the Anomaly

The simplest way to represent the solar anomaly in terms of an eccentric-circle theory, such as Hipparchos's, would be to engrave the Egyptian calendar scale off-centre from the zodiac, as in Figure 1. Both circles would be divided uniformly. (In the figure, for ease of comparison, we have divided each circle into 30° intervals. But the calendar

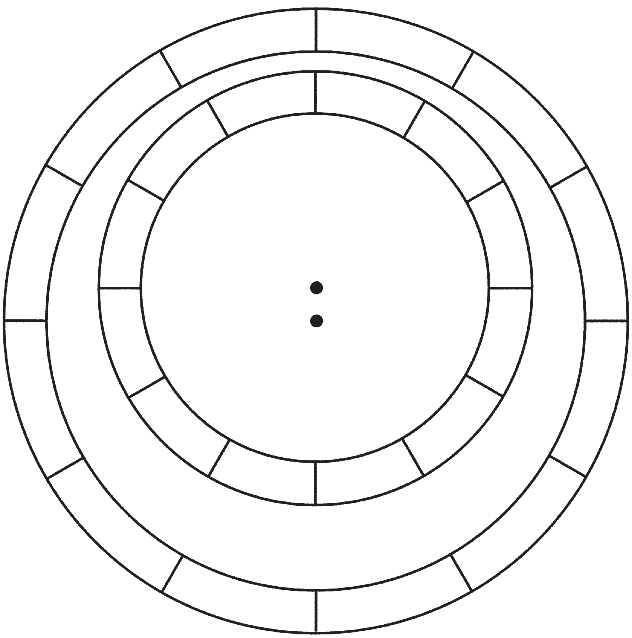

FIG. 1. Eccentric circles, both divided uniformly.

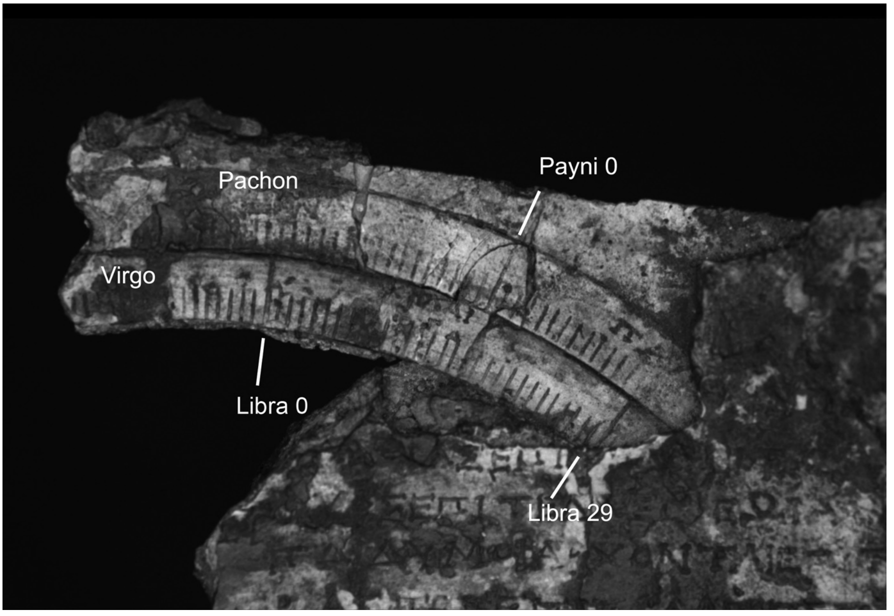

FIG. 2. A detail of Fragment C of the Antikythera mechanism. Copyright of the Antikythera Mechanism Research Project.

scale actually would have been divided into 365 days.) We have early attestation that off-centred, uniformly divided circles were used for graphically "computing" the longitude of the Sun, for, in his introduction to the *Handy tables*, Ptolemy briefly described just such a procedure.[9] In later Antiquity, Proklos explained how to make an instrument for finding the longitude of the Sun by engraving eccentric circles on a board or a bronze plate.[10] Moreover, a zodiac scale with eccentric calendar scale was a common feature on the backs of medieval European astrolabes.[11] Although the concentric zodiac was most commonly placed on the outside and the eccentric calendar scale on the inside, one could also do just the reverse. However, the maker of the Antikythera mechanism did not choose to use an eccentric circle: in Fragment C the two rings are clearly contiguous with one another. (See Figure 2.)

An alternative approach that some astrolabe makers found aesthetically pleasing was to display both circles (zodiac and calendar scale) concentrically, but to divide one of the circles non-uniformly. This style was favoured by Georg Hartmann at his Nuremberg workshop.[12] Usually, it was the calendar scale that was divided non-uniformly. That this need not be a tedious process is shown by Figure 3. The protractor used for dividing the circle is simply placed off-centre from the ring that is to be divided. It is thus no more trouble to divide a circle eccentrically than to divide it concentrically. In Figure 3, for simplicity's sake, we have drawn the fiducial marks (inside the annulus) to be directed toward the centre of division. In a more elegant version, points indicating the unequal division can be placed using the scheme shown in Figure 3, but the fiducial marks themselves could be pointed toward the geometrical centre of the circle.

Non-geometrical approaches to the solar anomaly should also be considered as possibilities. These would involve some sort of nonuniform division, but the construction would not be based on an eccentric circle model. For example, the division of the zodiac scale could be based on System A of the Babylonian solar theory, or a theory akin to it. Here, the Sun moves at a uniform angular speed of 30° per synodic month in the fast zone of the zodiac (which stretches from Virgo 13° to Pisces 27°) and 28°07′30″ per synodic month in the slow zone. One might therefore imagine a mechanician dividing the zodiac scale in such a way that the degree marks were spaced uniformly within each zone, but there would be discontinuities in the spacing of the marks at the zone boundaries. Since the entire preserved portion of the zodiac lies in the fast zone, we would not expect to see an obvious discontinuity in spacing, and we have no *a priori* grounds to reject this possibility.

Another possibility is a division based on season lengths. It is a common feature of Greek texts of the Hellenistic and Roman periods to mention the unequal lengths of the four seasons. In this case, the mechanician might divide the zodiac scale so that the degree marks are uniformly spaced within each season, but the spacing would change abruptly from one season to the next. But then one would expect to see an abrupt 5% change in the average spacing between degree marks at the autumn equinox,

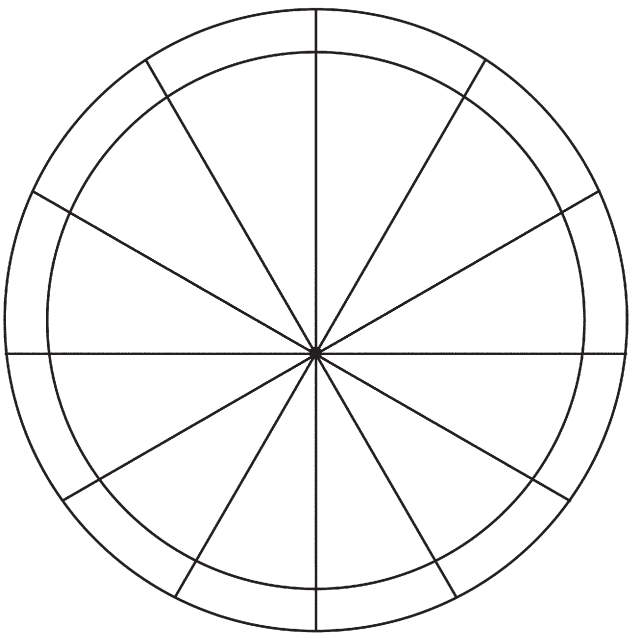

FIG. 3. A circle divided nonuniformly, by means of an off-centre, uniformly divided protractor.

which is not supported by the evidence. A more detailed investigation, too lengthy to describe here, shows that this hypothesis (at least if based on Hipparchos's season lengths) does not fit the actual spacing of the marks as well as other possibilities do. We will not consider this possibility any further. In what follows, we take System A as the prime representative of a non-geometrical, nonuniform division.

The question naturally arises whether the zodiac circle and the Egyptian calendar scale on the front of the Antikythera mechanism were both divided uniformly throughout their circles. Price noted that there seemed to be an inconsistency, which he found by comparing the divisions on the parts of the two circles that were visible to him. Noting that the marks on the two scales did not quite line up as one would expect if both were uniformly divided, he concluded, "Clearly, non-uniform division, probably of both scales, and probably unintentional, is the reason".[13] But he did not follow up on this.

In Figure 2 there is already evidence that some sort of nonuniform division was intended. For example, the first 29° of Libra[14] (Libra 0 to Libra 29) correspond to the interval from day $13\frac{1}{2}$ of Pachon to day 12 of Payni — an interval of $28\frac{1}{2}$ days. That is, the number of degrees (29) *exceeds* the number of days ($28\frac{1}{2}$). As there are 360 degrees in the circle and 365 days in the Egyptian year, if the circles were concentric and uniformly divided, one should have just the reverse: one would expect 29º to correspond to 29.4 days.[15]

Method of Investigation

While a stretch of about 44º of the zodiac and Egyptian calendar scales is visible, another 25º lies hidden behind a plate that covers the corroded remains. Using x-ray images to expand the range, we have made a careful investigation. The key result is that the two circular scales for the zodiac and the Egyptian calendar were indeed divided differently from one another, and quite deliberately so. Moreover, we shall demonstrate that the nonuniformity of division is of just the right character to be a good representation of the Sun's actual nonuniform motion. Thus, it turns out that a single solar pointer indicated both the position of the mean Sun (by showing the date on the Egyptian calendar scale) and the position of the true Sun (by showing the Sun's place in the zodiac). This is a simpler solution of the problem of the solar anomaly than others proposed so far for the Antikythera mechanism. Finally, as we shall see, it poses no difficulties for the lunar display as already understood.

Our investigation was based on a set of x-ray computed tomography (CT) images provided by the Antikythera Mechanism Research Project. The image set consisted of 51 x-ray "slices" spaced at 0.1 mm depth intervals. Using Photoshop software, we built up mosaic, composite views, each created from many dozens of layers. The composite in Figure 4 shows the zodiac marks (inner ring) and nearly all of the calendar marks (outer ring) over the preserved portions of the two scales, including the portions hidden from direct view. In Figure 4, the zodiac marks are labelled with their longitudes. For example, longitude 210 is the beginning of the sign of Scorpius.

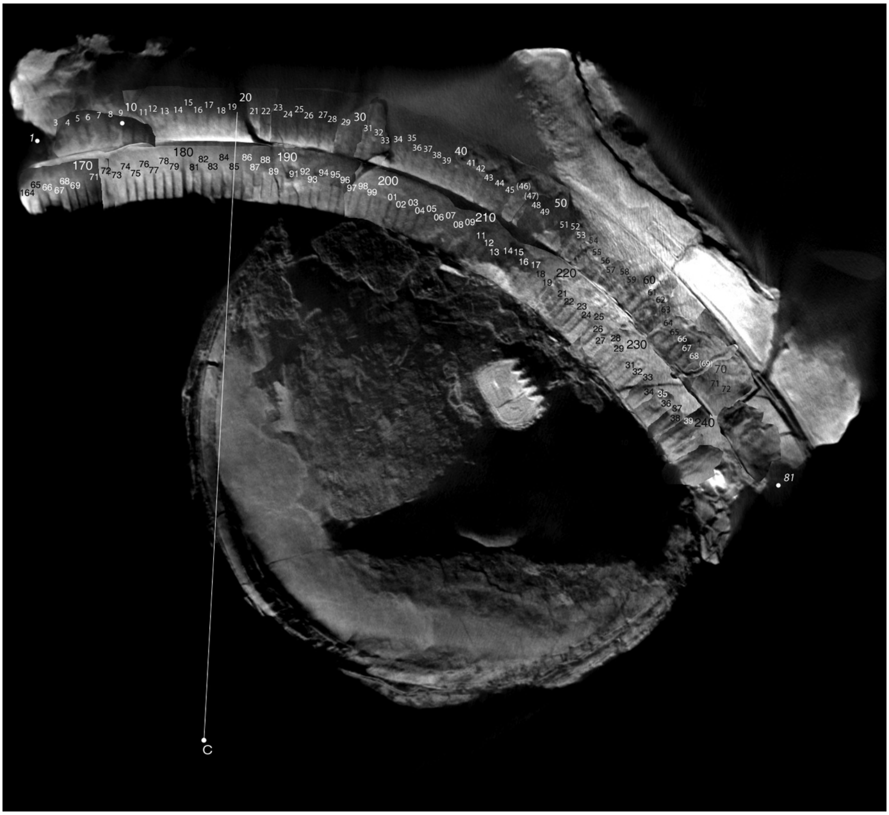

FIG. 4. Fragment C of the Antikythera mechanism. Composite radiograph produced in Photoshop from x-ray slices provided by the Antikythera Mechanism Research Project. On the zodiac scale, the numbers are degrees of longitude. The day marks on the Egyptian calendar scale have been numbered for reference. Copyright of the Antikythera Mechanism Research Project.

As is known, the Egyptian calendar ring is rotated out of its correct relation to the zodiac (for the date of the Antikythera mechanism) by about a third of a circle. (Perhaps someone on the ancient ship who was unversed in astronomy had played with the mechanism.) So, we do not deal with the absolute date in the Egyptian calendar, but instead simply number the day marks from 3 to 72. Again, it should be noted that Figure 4 is a composite and that no single x-ray slice shows all the relevant locations at one go. Not all of the calendar marks show up in the x-rays. Therefore, we also provide Figure 5, in which a properly scaled and oriented surface photograph is superposed on the composite x-ray image of Figure 4.

Beneath the Egyptian calendar scale, there are holes drilled at more or less regular intervals, which show up very clearly in the radiographs. The composite shown in Figure 6 was based on the same set of images, but incorporates some deeper slices

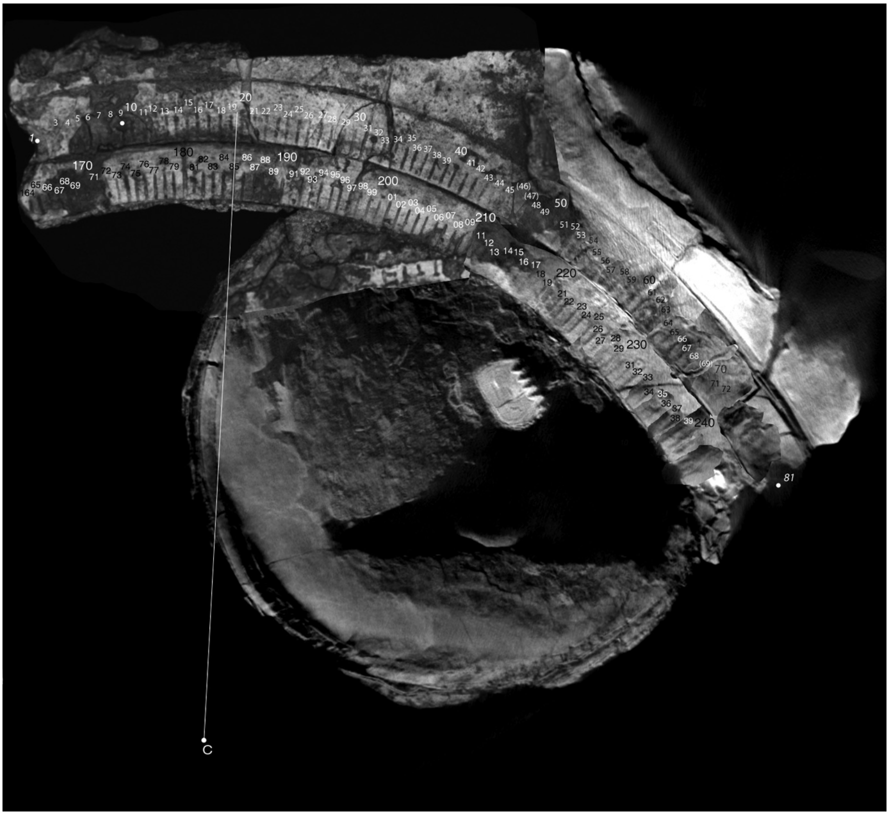

FIG. 5. Fragment C. Composite radiograph with a superimposed surface photograph of the visible portions of the zodiac and Egyptian calendar scales. Copyright of the Antikythera Mechanism Research Project.

to show the holes that lie behind the calendar scale. The calendar scale itself probably carried several small posts on its back that could be fitted into holes. In the operation of the device, the user was expected to set the Egyptian calendar into the correct relation with the zodiac for the date in question.[16] (The Egyptian calendar year was 365 days long, with no leap day, so the Egyptian year steadily shifts with respect to the seasons.) In Figure 6, the holes are numbered from 1 to 81, which are the first and last holes visible. The holes are numbered so that a hole of a given number lies reasonably near the corresponding day mark on the calendar scale, and in the case of number 9 the correspondence is exact: hole number 9 lies directly beneath calendar day mark 9. To allow cross-reference, holes 1, 9 and 81 are also labelled in Figures 4 and 5.

In Photoshop, one can place cross-hairs over points of an image and read off *x*- and *y*-coordinates, which we did for holes 1–73. (No calendar day marks are legible

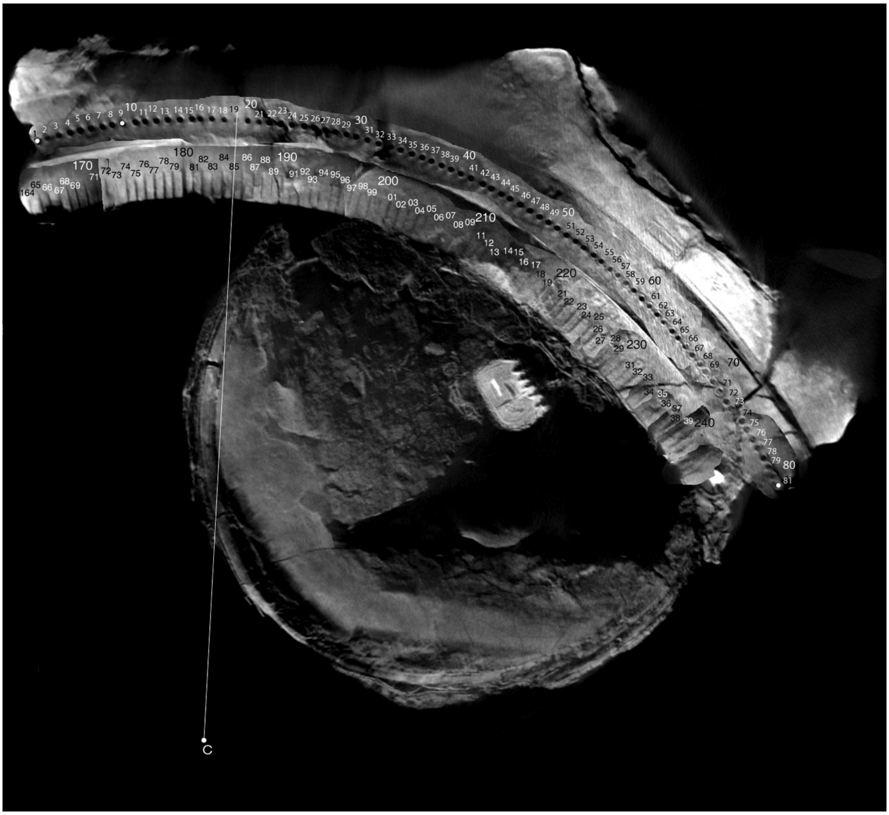

FIG. 6. Fragment C, showing degrees of longitude on the zodiac scale, as well as the holes underneath the calendar scale, which have been numbered for reference. Copyright of the Antikythera Mechanism Research Project.

beyond hole 73; moreover, there is an obvious fracture between holes 73 and 74.) We located the centre of the image of the circle of holes by a least-squares fit, and this centre is marked C in Figure 6. The centre C in Figures 4 and 5 is, similarly, the centre of the circle of holes.

An alternative way to find the centre of the rings is to use the circular boundary between the rings. We constructed another composite x-ray view (not reproduced here), showing more clearly the boundary between the zodiac and Egyptian calendar scales. On this composite, we took *x*- and *y*-coordinates of 134 points along the border between the two scales, and found the centre of this circle by a least-squares fit. The centre of the circular boundary between the two scales differs from the centre of the circle of holes by only a third of a millimetre; so from now on we take the centre of the circle of holes as a good representation of the centre of the ring system.

Consider Figure 4. Reliable data can be taken from about Virgo 19° (longitude 169) to about Scorpius 28° (longitude 238): the useable portion of the zodiac is shown in Figure 7. Applying a straight edge at C of Figure 4, one can easily read off which day number corresponds to a given degree of the zodiac. For example, in Figure 4, longitude 185 corresponds to a point between day 19 and day 20. A fractional day number can be noted, in this case about 19.6. To standardize the interpolation process, we drew a circle with centre C, slightly outside the inner edge of the zodiac circle — just far enough beyond the edge to guarantee that the circle crosses nearly all of the degree marks. The *x*- and *y*-coordinates were taken of the point where each zodiac degree crosses this reference circle. Similarly, we drew a second circle to facilitate taking the *x*- and *y*-coordinates of the day marks on the Egyptian calendar ring. Then we calculated the angular positions of the zodiac marks, as viewed from C, and compared them with the angular positions of the day marks, to determine the fractional part of the day number.

The results of direct measurement are shown in the first two columns of Table 1. Column λ gives the longitudes of marks engraved on the zodiac.[17] For convenience in the analysis, we tabulate the data from the point nearest the solar perigee, where the equation of centre is most nearly zero; that is, we start at longitude 238, and reckon down to longitude 169. Column t gives the day number corresponding to each degree mark. For example, longitude 238 corresponds to day mark 71.65, and longitude 169 corresponds to day mark 3.76. Again, the integral part of each day reading is obtained by simple *counting*; only the fractional part of the day number is obtained by measurement and interpolation. As mentioned above, these data can

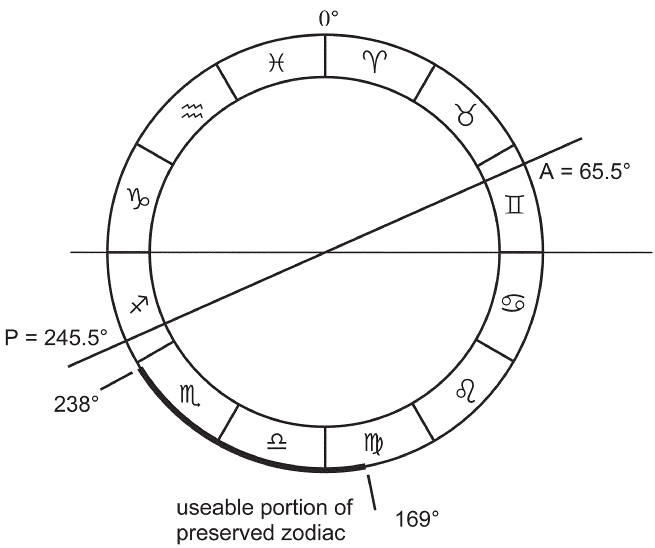

FIG. 7. The useable portion of the zodiac, displayed with the apogee and perigee of Hipparchos's solar theory. "Usability" requires preservation of the degree marks on the zodiac scale, as well as of a nearly continuous sequence of day marks on the calendar scale.

TABLE 1. Correspondence of longitude marks on the zodiac scale and day marks on the Egyptian calendar scale. The data in column 2 have been taken from Figure 4, unless printed in italic type, in which case they have been taken from Figure 5.

		Δt (days)		
λ (°)	*t* (days)	Antikythera mechanism	For uniform division	For Hipparchos
238	71.65	0.00	0.00	0.00
237	70.75	0.91	1.01	0.97
236	70.01	1.64	2.03	1.95
235			3.04	2.92
234	68.02	3.64	4.06	3.89
233	67.05	4.61	5.07	4.86
232	65.79	5.86	6.09	5.84
231	64.89	6.76	7.10	6.81
230	63.83	7.82	8.12	7.79
229	62.85	8.80	9.13	8.76
228	61.80	9.86	10.15	9.73
227	60.81	10.85	11.16	10.71
226	59.81	11.84	12.18	11.68
225	59.06	12.60	13.19	12.66
224	57.74	13.91	14.20	13.63
223	56.80	14.86	15.22	14.61
222	55.69	15.96	16.23	15.58
221	54.81	16.84	17.25	16.56
220	53.93	17.72	18.26	17.54
219	52.99	18.66	19.28	18.51
218	51.78	19.87	20.29	19.49
217	50.92	20.74	21.31	20.47
216	49.98	21.67	22.32	21.44
215	49.04	22.61	23.34	22.42
214	48.08	23.57	24.35	23.40
213			25.36	24.38
212			26.38	25.36
211	45.00	26.65	27.39	26.34
210	44.07	27.58	28.41	27.32
209	43.27	28.39	29.42	28.30
208	42.43	29.22	30.44	29.28
207	41.08	30.58	31.45	30.26
206	40.17	31.48	32.47	31.24
205	39.01	32.64	33.48	32.22
204	38.20	33.45	34.50	33.21
203	37.28	34.38	35.51	34.19
202	36.28	35.37	36.53	35.17
201	35.03	36.62	37.54	36.16
200	34.01	37.64	38.55	37.14
199	33.23	38.43	39.57	38.13
198	31.97	39.68	40.58	39.11
197	31.09	40.57	41.60	40.10
196	*30.00*	*41.66*	42.61	41.09
195	*29.13*	*42.53*	43.63	42.07
194	*28.14*	*43.52*	44.64	43.06
193	*27.21*	*44.44*	45.66	44.05
192	*26.36*	*45.29*	46.67	45.04
191	25.39	46.26	47.69	46.03
190	*24.36*	*47.29*	48.70	47.02
189	*23.13*	*48.52*	49.71	48.01
188	*22.19*	*49.46*	50.73	49.00
187	*21.44*	*50.21*	51.74	49.99

Table 1 cont'd]

186	20.44	51.21	52.76	50.99
185	19.56	52.09	53.77	51.98
184	18.52	53.14	54.79	52.97
183	17.45	54.20	55.80	53.97
182	16.43	55.22	56.82	54.96
181	15.60	56.06	57.83	55.96
180	14.68	56.97	58.85	56.96
179	13.46	58.19	59.86	57.95
178	12.30	59.35	60.88	58.95
177	11.23	60.42	61.89	59.95
176	10.38	61.27	62.90	60.95
175	9.50	62.15	63.92	61.95
174	8.70	62.95	64.93	62.95
173	7.90	63.75	65.95	63.95
172	6.88	64.77	66.96	64.96
171	5.82	65.83	67.98	65.96
170	4.69	66.96	68.99	66.96
169	3.76	67.89	70.01	67.97

be checked directly (to the nearest tenth day) on Figure 4. (Data to the hundredth of a day come from the computerized process described above.) The third column displays the time intervals Δt, counting from the day when the longitude was 238. Thus the third column is obtained by subtracting each entry in column 2 from 71.65. For example, the 69° of motion from longitude 238 to longitude 169 corresponds to 67.89 days on the Antikythera mechanism.

A single x-ray composite (Figure 4) has been used to obtain the great bulk of the day numbers (column t in Table 1), thus guaranteeing the greatest possible consistency. However, the day numbers corresponding to longitude marks 187–190 and 192–196 were determined with the aid of the surface photograph of Figure 5. This is because day marks 22–24 and 27–29, although visible in the x-ray images, were somewhat clearer in the surface photograph. The nine values based on the surface photograph are printed in italic type in column t. But use of Figure 4 would not have resulted in substantially different values.

There are two gaps in Table 1, one at longitude 235 and the other at 212–213, because the corresponding day marks can be seen neither on the radiographs nor on the surface photograph. One day mark (day 69) is missing in the first gap, because of a crack in the metal, and two day marks (days 46–47) are missing in the second. Since our method involves counting days, it is essential to know for sure how many day marks are missing because of illegibility. Fortunately, for these two gaps there is no doubt at all. Consider first the gap at day marks 46–47. The long marks 31 and 61 are the beginning and ending of the month of Payni, and must therefore be separated by an interval of 30 days, which is consistent with inserting two day marks. Moreover, the distance between marks 45 and 48 is consistent with the insertion of two marks. Consider now day mark 69. The distance between marks 68 and 70 is 2.48 mm (measured along the guiding circle we used for taking coordinates, very close to the circle of holes) and the average distance between two marks is 1.28 mm, so it is clear that there is one and only one mark between 68 and 70.[18]

Analysis of the Longitude and Day-Number Data

The basic question is how λ and "Δ*t* Antikythera" are related. The graphical relation between λ and Δ*t* is shown in Figure 8, where the data obtained by comparing the day marks on the Egyptian calendar ring with the degree marks on the zodiac ring are represented by small black triangles.

If the two concentric scales were both divided uniformly (as has been supposed by most workers until now), then a change in true longitude Δλ on the zodiac scale should correspond to an equal change in mean longitude. Thus, the expected time interval on the calendar scale would be given by simple conversion of degrees into days. Take, for example, longitude 169. Thus, one would expect a time interval of

$$\begin{aligned}\Delta t &= \Delta\lambda \times (365.25 \text{ days} / 360°)\\ &= (238 - 169) \times (365.25/360)\\ &= 70.01 \text{ days},\end{aligned}$$

as shown in Table 1, in the column headed "Δ*t* For uniform division". In Figure 8, the time intervals for a uniformly-moving Sun are shown by the solid line.

As we have seen, a change in the Sun's longitude of 69° on this part of the zodiac scale corresponds to a time interval of 67.89 days on the Antikythera mechanism; but

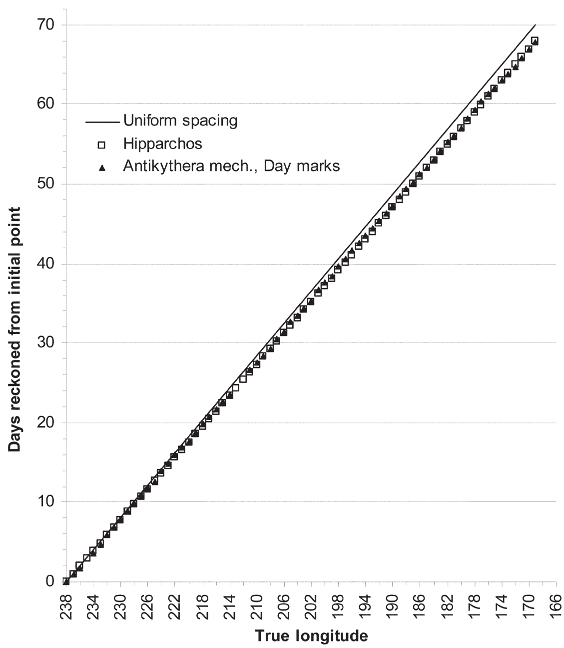

FIG. 8. Graphical display of the three final columns of Table 1. The horizontal axis is true longitude of the Sun, reckoned from the vernal equinox. The vertical axis is the time interval required for the Sun to run from longitude 238 to the plotted longitudes. This graph is based on a comparison of the degree marks on the zodiac scale with the day marks on the Egyptian calendar scale.

if the Sun were represented as moving uniformly, it should take 70.01 days. This is a discrepancy of 2.1 days, which is too large to be accounted for by accidental causes (which might account for a difference of half a day). It is clear that the hypothesis of uniform division (column 4) cannot be retained.

We are both lucky and unlucky in the portion of the zodiac that is preserved. We are lucky to have an interval from near perigee (where the equation of centre is nearly zero in Hipparchos's solar theory) to a place near the quadrant of the eccentric (where the equation of centre is nearly a maximum). (We shall explain the unluckiness below.) Moreover, the discrepancy between time intervals and displacement in longitude is about what one would expect (in both magnitude as well as sign) if the division of the scales was meant to reflect Hipparchos's eccentric-circle theory, or some other theory based on comparable numerical parameters.

Let us therefore make an explicit comparison. In Hipparchos's solar theory, the longitude of the apogee is $A = 65.5°$ (Figure 7) and the eccentricity of the Sun's circle is $e = 1/24 \approx 0.0417$. The relation between the mean longitude $\bar{\lambda}$ and the true longitude λ is

$$\bar{\lambda} = \lambda - q,$$

where q is the equation of centre, given by[19]

$$\sin q = -\, e \sin (\lambda - A).$$

For the starting value of $\lambda = 238°$, we have

$$\begin{aligned} \sin q &= -\,0.0417 \sin (238° - 65.5°)\,. \\ q &= -\,0.31°. \\ \bar{\lambda} &= 238.00° + 0.31° \\ &= 238.31°, \end{aligned}$$

which is the value of the mean longitude corresponding to true longitude 238° in Hipparchos's theory. Then for each line in Table 1 we perform a similar calculation. Take, for example, the last line, for which $\lambda = 169°$:

$$\begin{aligned} \sin q &= -\,0.0417 \sin (169° - 65.5°). \\ q &= -\,2.32°. \\ \bar{\lambda} &= 169.00° + 2.32° \\ &= 171.32°. \end{aligned}$$

The time interval $\Delta\lambda$ in the Hipparchos column of Table 1 opposite $\lambda = 169°$ is

$$\begin{aligned} \Delta t_{169} &= (\bar{\lambda}_{238} - \bar{\lambda}_{169}) \times (365.25 \text{ days} / 360°) \\ &= (238.31° - 171.32°) \times 1.01458 \text{ days/}° \\ &= 67.97 \text{ days}, \end{aligned}$$

which appears in the last line of the right column. To sum up:

Corresponding to a true longitude interval (from 238° to 169°) of $\Delta\lambda = 69.00°$, uniformly divided, concentric circles would imply a time interval of $\Delta t = 70.01^{d}$,

while Hipparchos's theory would require a time interval of $\Delta t = 67.97^{d}$,
and we actually find on the instrument a time interval of $\Delta t = 67.89^{d}$.

The remainder of Table 1 is filled out in the same way.

In Table 1, one can compare columns 3 and 5 (for the Antikythera mechanism and for Hipparchos's theory, respectively), and the correspondence is rather good. Exact agreements should not be given too much importance, since the nature of the data and our methods of measurement suggest that errors could occasionally amount to several tenths of a day. (For example, the positions of some of the degree marks appear to shift by a non-negligible fraction of a degree when one moves from one x-ray slice to others.) But the general trend is quite satisfactory. See Figure 8, in which the Antikythera data (black marks) are compared with Hipparchos's theory (open squares). As the graph shows, the Antikythera mechanism does a good job of displaying the nonuniform motion of the Sun, simply by means of an appropriately nonuniform division of the scales. No extra epicyclic gearing (as has been postulated by other researchers) is required.

We performed a least-squares fit of an eccentric circle theory (such as Hipparchos's) to the Antikythera mechanism time intervals of column 3, holding the longitude of the apogee fixed at 65.5°, and varying the eccentricity, with a best-fit result of e = 0.0357, where, again, the Hipparchos value is 0.0417. (That is, we minimized the sum of the squares of the differences of columns 3 and 5 in Table 1, omitting the cases that are blank in column 3.[20]) At the scale of the Antikythera mechanism (inner radius of zodiac = 6.25 cm), the difference (0.0417 – 0.0357) corresponds to an error in centre placement of less than four tenths of a millimetre. Whether the maker of the mechanism was aiming at Hipparchos's eccentricity or really did intend a smaller value, it is not possible to say.

The method of taking data described here is very robust. The x-ray slices resulting from the CT technology came to us automatically scaled, oriented and positioned properly with respect to one another, so composites could be built up in Photoshop with negligible placement error. Moreover, separate workers, working on separate computers, confirm the placement of the individual marks with discrepancies of less than a tenth of a day. Finally, the method is relatively insensitive to error in the placement of the centre. Because the inner radius of the zodiac is 6.25 cm, while the fiducial edges of the zodiac and calendar scales are separated by only 0.75 cm, there is an eight-fold reduction in centre error: for example, an error of a millimetre in the location of centre C (a larger error than is likely) would translate into a maximum error in the reading of a day number on the calendar scale of only 0.12 mm.

Nonuniform Division of the Zodiac

If the zodiac scale is divided nonuniformly, one might hope to be able to measure, in millimetres, say, the lengths of separate zodiac arcs to detect the nonuniformity directly. But we are unlucky in the portion of the zodiac that is preserved, as we have less than one quadrant, all located near to and on one side of the perigee. With a solar apogee and eccentricity in agreement with Hipparchos's theory, a zodiac of radius 62.5 mm, and the angular division made according to the scheme of Figure 3, one would expect the length of the arc 168 → 203 to exceed the length of the arc 203 → 238 (both being arcs of 35°) by only six tenths of a millimetre. And the chords of the two arcs, which would be more susceptible to direct measurement, would differ by a bit less. Unfortunately, the corroded arc bearing the zodiac and Egyptian calendars scales does not lie in a plane, but has a wavy shape. Moreover, this arc was accidentally broken off and glued back onto the main fragment during the twentieth century. This principal break seems to be that near day mark 32 in Figure 5. There is another break at day 20. In the x-rays, other fractures can be seen, for example, that in Figure 6 near longitude 232, as well as another near hole 73, which causes a small but abrupt change in the direction of the curve fitted to the holes. If we had a 35º arc straddling the perigee and a 35º arc straddling the apogee, these would be expected to differ in length by fully 3 mm. But, with the portion of the zodiac that we actually have, direct comparison of arcs cannot really be used to confirm or refute the nonuniform division. One must therefore rely on angles and times accumulated over the whole extant portion of the zodiac, as described above, and as illustrated in Figure 4. To put it another way, because the two scales have remained contiguous, cracks and warps do not affect very strongly our method based on a simple matching of day marks against the corresponding degree marks.

If deliberately nonuniform division is taken as demonstrated, *which* of the scales is divided nonuniformly? As mentioned above, the calendar scale was intended to be rotated by hand by one day every four years. It would not make sense to divide the calendar scale unequally, since the movement of the calendar scale to a new location would effectively shift the location of the apogee (which is fixed in Hipparchos's theory).[21] Given the design choice of concentric circles and a moveable Egyptian calendar ring, the only option available to the maker of the Antikythera mechanism was to divide the zodiac ring nonuniformly.

Some moderns may find it surprising or implausible to divide the zodiac nonuniformly. But it suffices to recall that on the rete of an astrolabe, as well as on the sky disk of an anaphoric clock, the zodiac was divided nonuniformly (albeit by a different technique).[22] Thus, an ancient user — at least one adept enough at astronomy to use the Antikythera mechanism — would not have been put off by a nonuniformly divided zodiac. And here it may also help to point out that the mechanism was not intended as a pictorial representation, literal and realistic in every way. The displays of abstract luni-solar cycles and the four-year cycle of athletic contests can serve to remind us of that.

Fine Structure of the Solar Anomaly

Let us now examine the structure of the nonuniformity more closely. This we do by subtracting out the mean motion of the Sun so that the anomaly is seen in isolation.

When the Sun's mean longitude changes by $\Delta\bar{\lambda}$ and its true longitude changes by $\Delta\lambda$, the equation of centre will change by

$$\begin{aligned}\Delta q &= \Delta\lambda - \Delta\bar{\lambda} \\ &= \Delta\lambda - (360^{\circ}/365.25^{d})\,\Delta t.\end{aligned}$$

Figure 9 is therefore constructed in the following way. The vertical coordinates of the points labelled "Antikythera mechanism" are calculated from Table 1 as

$$(238 - \text{column 1}) - (360^{\circ}/365.25^{d})\,(71.65 - \text{column 2}).$$

For example, for the point at longitude 208, the vertical coordinate is (238 – 208) – (360/365.25) (71.65 – 42.43) = 1.20°. This represents the excess of Sun's displacement from the starting longitude of 238 over what the displacement would have been if the Sun had moved at the uniform rate of 360° in 365.25 days. To put it another way, the vertical coordinate of the graph gives the change in the Sun's equation of centre from the value of this equation at longitude 238.

Also plotted on Figure 9 is the equation of centre for a Hipparchos-style eccentric model with the apogee at longitude 65.5°, for three different eccentricities: Hipparchos's (e = 0.0417), the best-fit model (e = 0.0357), and a similarly offset smaller value (e = 0.0297). (In each case, it is not precisely the equation of centre that is plotted, but the excess of the equation of centre over the value of the equation at longitude 238.) Finally, we have plotted the "equation of centre" for the Babylonian solar theory of System A. This straight line shows the way the excess of the Sun's longitude over its mean longitude would develop, at a solar motion of 30° per synodic month,

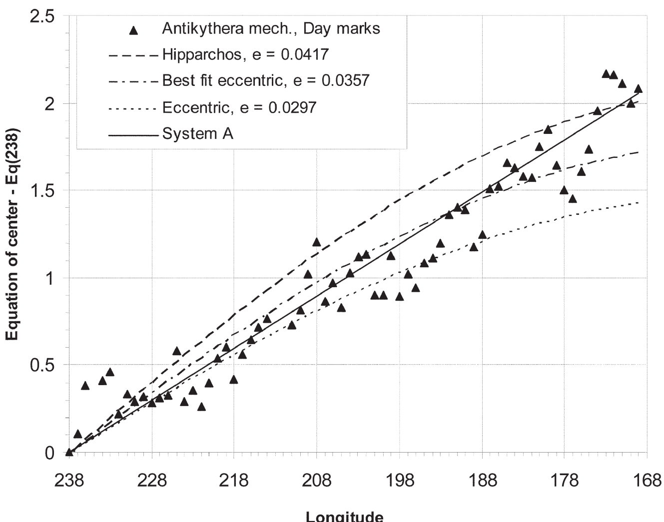

FIG. 9. Comparison of the solar anomaly built into the zodiac and Egyptian calendar scales of the Antikythera mechanism with several possible underlying theories. The vertical axis is the difference between the Sun's equation of centre at the given longitude and its equation of centre at longitude 238.

consistent with the Sun being in the fast zone of the Babylonian theory. We use 29.53 days for the synodic month, a good approximation to both Greek and Babylonian month values for the epoch of the mechanism. Figure 9 represents quite a stringent test. We start the Antikythera mechanism and the theoretical models off together at longitude 238, and see how the equations of centre develop over the next 69º.

Consider first the eccentric circle models. The Hipparchian value of the eccentricity is clearly too high, and the symmetrically placed lower value of 0.0297 is clearly too low. The data are well matched by the best-fit eccentricity of 0.0357. But the non-geometric model (System A) clearly cannot be excluded. Indeed, if we compare the sum of the squared residuals for System A with the sum of the squared residuals for the best-fit eccentric model, we find that System A does somewhat better (1.66 deg^2 for System A, and 2.64 deg^2 for the eccentric with $e = 0.0357$). (The residuals for the eccentric model can be reduced by reducing the longitude of the apogee. For example, with the apogee shifted to 40º, the best-fit eccentricity becomes about 0.030 and the residuals become nearly, but not quite, indistinguishable from those of System A — though this seems too large an apogee shift to be seriously considered.) The good performance of System A corresponds to the overall visual impression of linearity in Figure 9. However, the winning margin for System A is largely due to the single clump of six points at the extreme upper right. Indeed, with the last six points excluded, the best-fit eccentric actually beats System A by a small margin. To sum up: while the numbers do somewhat favour System A over the eccentric circle model, we cannot exclude either based on the short portion of the zodiac that is preserved.[23] This is not surprising when we recall that the two models are a good match to each other in the case of a small eccentricity like the Sun's.

The more important conclusion is that the Antikythera mechanism did indeed account for the anomalous motion of the Sun by nonuniform division of the zodiac scale. For if the scales represented a uniformly-moving Sun, Figure 9 would be a horizontal straight line, which it very clearly is not. The possibility that the agreement shown in Figure 9 is due to chance is almost completely excluded. The agreement of the data with the eccentric-circle theory is quite good. Here, of course, we do have an adjustable parameter in the eccentricity. But the possibility that the data represent only accidental errors and that the data, simply by chance, happen to agree in magnitude and phase with an eccentric-circle theory with the correct apogee and a plausible eccentricity seems extremely remote. Even more telling is the agreement of the mechanism's scales with System A, for *the line representing System A has no adjustable parameters at all*. We start off at longitude 238 and examine the change in the mechanism's equation of centre over the next 69°. The plotted points come directly from comparison of the graduations on the zodiac and Egyptian calendar scales, and the slope of the straight line is determined by System A.

While the data do not permit us to choose decisively between the eccentrically placed protractor of Figure 3 and a scheme, such as System A, that is based on a piecewise-uniform division within zones, an argument can be made for the geometrical scheme. On the Antikythera mechanism, the lunar pointer must share the

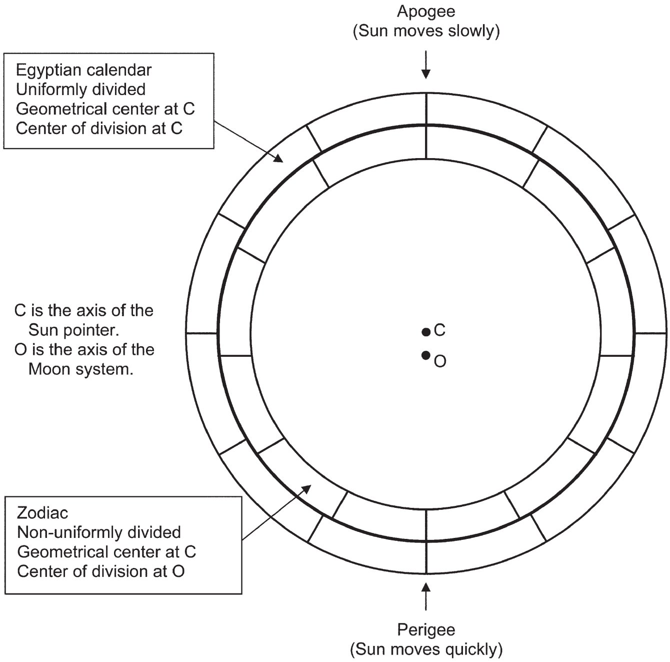

FIG. 10. Our reconstruction of the function of the zodiac and Egyptian calendar scales, including the placement of the axle of the lunar mechanism and the axis of rotation of the Sun pointer.

same zodiac as the solar pointer. A piecewise-uniform division of the zodiac by zones would corrupt the zodiac for the display of the Moon's position. In contrast, if we adopt a nonuniform division of the zodiac by means of an eccentrically-placed protractor, this does not cause any difficulty for the mechanism's lunar apparatus as already understood. Refer to Figure 10. The Egyptian calendar scale and the zodiac both have their geometrical centres at point C. In this reconstruction, the calendar scale is divided uniformly with respect to its own centre C. But the zodiac (while centred at C) is divided nonuniformly, by placing the centre of a protractor at O. In thinking of the system as a representation of Hipparchos's model, one should regard O as the Earth, and C as the centre of the Sun's eccentric circle. The axis of rotation of the Sun pointer must therefore pass through C. But the axle of the lunar apparatus must pass through O. It is true that the zodiac ring is off-centre from O — but the degree marks are laid out *along the zodiac at uniform angular intervals as viewed from O*. Thus, the zodiac ring functions completely normally as far as the lunar apparatus is concerned.

Gearing

Let us take up the question of the necessary gearing. The inner radius of the zodiac of the Antikythera mechanism is r_1 = 62.5 mm and the outer radius is r_2 = 70.0

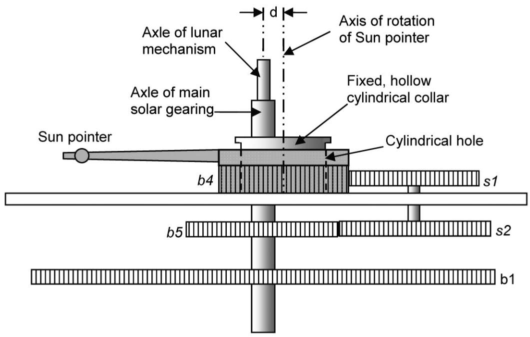

FIG. 11. A possible mechanism for effecting the off-centredness of the axis of rotation of the Sun pointer. The axis is displaced by distance *d* from the main solar (and lunar) axis. In the Antikythera mechanism, *d* is about 2.6 to 3 mm. Only a small portion of b1 is shown.

mm.[24] With the eccentricity $e = 0.0417$ in Hipparchos's theory, the corresponding off-centredness of the physical axes would be $d = 0.0417 \times 62.5$ mm = 2.6 mm if we use the inner radius r_1 (which we judge most likely to have corresponded to the fiducial edge of the scale). If we use the outer radius r_2 we would get $d = 2.9$ mm. How might gearing have been contrived to give two axes of rotation only 3 mm apart?

Here, of course, we can only conjecture, and there is always more than one way to do it. But a suggested mechanism is shown in Figure 11. The (white) plate, gear b1, the axles of the main solar gearing and of the lunar mechanism, are exactly as in the most recent reconstruction.[25] We propose four new gears and a fixed cylindrical collar. The collar is rigidly attached to the plate. The axles pass through the hollow of the collar, but they are off-centre by a distance *d* from the collar's axis of symmetry. Gear b5 turns with the axle of main solar gearing, and it engages s2. Fixed to axle s is s1, with fewer teeth than s2. s1 engages b4; and b4 is free to turn about the cylindrical collar. On b4 is mounted the Sun pointer. It results that the Sun pointer turns uniformly and at the same rate as the main solar axle, but about an off-centre axis. The hollow cylinder must have a flange or tabs, as shown, to keep b4 from sliding out of place.

One of us (AST) actually built brass solar gearing on a scale comparable to that of the Antikythera mechanism. See Figure 12. The plate is made of transparent plastic, so the two solar gears below it (s2 and b5) can be faintly seen, along with the main drive gear b1. Above the plate can be seen s1 and b4, along with the fixed cylindrical collar. The gearing for the lunar phase display, as originally proposed by Michael Wright,[26] can function without any modification. (The black pointer at the top of the photograph is unrelated to the solar and lunar displays.)

The specifications for the new gearing are that it not change the rotation rate of the Sun and that it offset the rotation axis of the sun pointer by *d* = about 3 mm. The first condition requires the number of teeth to satisfy (b5/s2)(s1/b4) = 1. (The names of the gears stand for their tooth numbers.) The second requires $m(\text{b5} + \text{s2}) = 2d + m(\text{b4} + \text{s1})$, where m is the gear module, i.e., the gear diameter per tooth. The

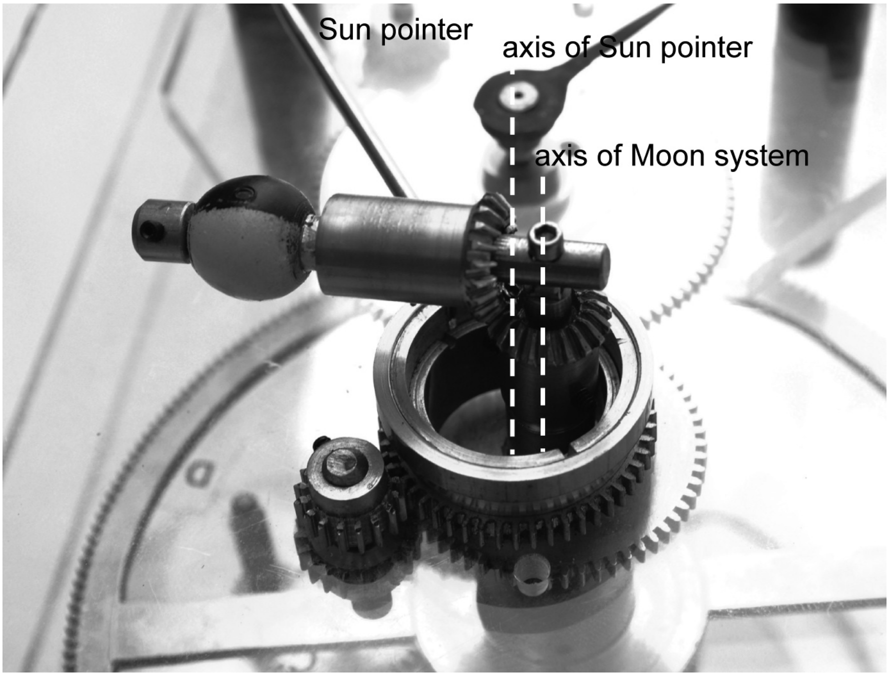

FIG. 12. A mechanical model of the proposed solar gears and cylindrical collar.

diagram of Figure 11 was drawn assuming module $m = 0.5$ mm and choosing b5 = s2 = 40 teeth, and s1 = b4 = 34 teeth. The separation of the centres of b5 and s2 is then $m(20 + 20) = 20$ mm, and the separation between centres of s1 and b4 is $m(17 + 17) = 17$ mm. Since s1 and s2 share the same axle, b4 and b5 are offset by 3 mm. The inside dimension of the cylindrical collar has to be large enough to accommodate the axle of the main solar gearing and the offset d. If necessary, one could make the gears a little bigger; e.g., one could use b5, s2, s1, b4 = 60, 20, 17, 51. This would increase the diameter of b4 to 25.5 mm and allow the cylindrical collar to increase as well. And these are the choices behind the model in Figure 12.

Summary and Conclusions

Figure 9 demonstrates that the solar anomaly was accounted for and displayed on the Antikythera mechanism simply by nonuniform division of one of the scales, most likely of the zodiac ring. The correspondence between time intervals and zodiacal arcs is of the right size, the right sign, and the right trend for the preserved portion of the zodiac. For the builder of the Antikythera mechanism, nonuniform division of the zodiac was a sensible design decision. The gears of the mechanism are not cut with high precision in the tooth spacing; and when we include the effects of backlash, it may have been hopeless to try to model a phenomenon of 2° amplitude by using epicyclic gearing. Nonuniform division of the zodiac is a simple solution that matches achievable precision with the smallness of the effect being modelled.

We are, however, still left with several alternatives. First of all, we do not know for certain whether the mechanism included some means of preventing the non-uniform division of the zodiac from disturbing the longitudes of the Moon. It is conceivable that the maker simply ignored this problem and left the Sun and Moon axes co-axial. In such a case, the lunar axle (as well as the solar axle) would pass through point C in Figure 10, and the gears of Figure 11 (for effecting an off-centredness of the solar and lunar axes) would not exist. But this would have meant allowing the solar solution to generate errors of up to 2° in the longitude of the Moon.[27] This seems to us an unlikely possibility, in view of the pains the maker took to model a 5° lunar anomaly by means of epicyclic gearing. We believe, therefore, that the solar and lunar axes were offset by about 3 mm, as in Figure 10, so that the longitudes of the Moon would not be corrupted by the nonuniform division of the zodiac. We have suggested the necessary gearing in Figure 11.

Second, we cannot be certain from the numerical data whether the division of the zodiac was based on an eccentric-circle theory or on a scheme involving zones of constant speed, such as the Babylonian System A. The data somewhat favour System A. But in either case, the offset between the solar and lunar axles would still be required. If System A is involved, then point O in Figure 10, where the Moon's axle passes, is simply the point from which the divisions of the zodiac look *nearly* uniformly spaced, *to a good enough accuracy*. If the eccentric protractor is involved, then O is the position from which the zodiac marks are *truly* equally spaced in angle.

It might seem a strange combination to use a division of the zodiac based on System A together with the geometrical offset of points C and O. But in the period of the Antikythera mechanism, we have instances of arithmetical and geometrical schemes happily coexisting. For example, Geminos (first century B.C.), in the first chapter of his *Introduction to the phenomena*, discusses the nature of the motion of celestial bodies, which he insists is uniform and circular, and he illustrates this with the eccentric-circle theory of the Sun; but in his chapter 18 he gives a detailed account of the Moon's *nonuniform* motion based on the Babylonian numerical parameters of System B. A second example is provided by the *Celestial teaching* of Leptines.[28] This papyrus of the second century B.C. provides a description of the circles on the celestial sphere, but its scheme for the length of the day is a simple arithmetic progression, according to which the day length changes by 1/1080 of a day for each of 180 days from one solstice to the next. In the period of Greek astronomy that we are considering, geometrical principles and arithmetical convenience can exist side-by-side, even in the same work. A combination of geometrically offset axles and a System-A division of the zodiac would, of course, still entail minor errors in the longitude of the Moon, though these would now be only a fraction of a degree. The data do favour System A. But the off-centre protractor, illustrated in Figures 3 and 10, is simple, not only in concept but also in terms of fabrication, and it involves no approximations at all.

As we have argued, whether the solar anomaly was modelled by System A or by the geometrical theory, a mechanism for offsetting the Sun and Moon axes, such as that presented in Figures 11 and 12, would have been required. The solar proposal of Figures 11 and 12 is very much in the spirit of the Antikythera mechanism. The four-gear system used to effect an offset solar axis has much in common with the four-gear system previously proposed as a representation of the lunar epicycle.[29] The solution is also economical. Note that if the maker had used eccentric but uniformly divided circles (as in Figure 1), he still would have had to separate the Sun pointer's axis of rotation from the axle of the lunar mechanism. So the maker's design choice of a nonuniformly divided zodiac did not introduce any extra mechanical complexity, and still allowed him to avoid the need for a second solar pointer to indicate the day of the year. An attractive feature of the solution is that the upper parts of the solar and lunar axles are unaffected and are still available for placing the gearing of the Moon phase display.[30]

2. THE PLANETARY DISPLAYS

The Nature of the Planetary Displays

It is possible that a significant portion of the gearwork of the Antikythera mechanism has been lost. In particular, much seems to be missing between the front plate and the large drive gear b1. There is a great contrast in the gear density of the front and back portions of the mechanism, which may be a sign that some of the gearing of the front part of the mechanism is missing. Inscriptions on the front cover and the back cover mention some of the planets by name. Most researchers are therefore agreed that the mechanism originally included some sort of planetary display, but its exact nature is difficult to determine. For, unfortunately, no planetary gears at all survive, except perhaps for a single wheel, r1, of 63 teeth, which has no assigned role in the solar, lunar, or calendrical trains.

Most proposals have focused on the possibility of kinematic displays, in which the progress in longitude of each planet was indicated by a pointer moving back and forth around the zodiac, according to the principles of deferent and epicycle theory. Generally, these proposals have postulated epicyclic gears riding on the large "main drive gear", b1. While this is of course possible, there are two difficulties with such a proposal. First of all, if b1 were intended to carry large numbers of gears, it is hard to see why it was formed as it is, with four large missing sectors. One might think that having the extra strength of a full disk — or at least having the extra space available for mounting all those gears — would have been desirable.

Second, on the assumption that the planets were handled by putting epicyclic gearing on b1, it is not possible to assign any astronomical reason for its tooth count of 223 or 224, as measured by Freeth *et al.* The teeth on b1 are substantially larger than those on the great majority of the other extant wheels of the mechanism. The module for the great majority of the extant gears averages about 0.50 mm/tooth, while the module of b1 is about 0.58 mm/tooth. The 16% larger-than-normal size of the teeth on b1 suggests that the designer of the mechanism, once he had decided on the overall diameter of the wheel, felt it was important to retain this particular number of 223 or 224. This is an indication that the tooth count of b1 met some important design criterion.

There are three additional reasons to remain open to a broader range of possibilities for the planetary displays. First, while the mechanism for the solar eccentricity described above is not incompatible with a kinematic planetary display, it would have made kinematic incorporation of the planets more complicated. Also, all of the included planets would presumably have had their mechanisms centred on the b axis. This means that their longitudes would all have been displayed on the nonuniformly-divided zodiac scale. Now, it is one thing to accommodate a single celestial body, the Moon, to a zodiac scale that was divided nonuniformly for the sake of the Sun. But it would seem an odd design choice to make all the planets suffer in the same way, when the maker could have chosen instead simply to divide both scales uniformly but to place the calendar scale off centre.

Second, we take the front cover inscription as suggesting the possibility that the mechanism did not include a display of the zodiacal motion of Venus or the other planets. To be sure, the front cover inscription probably mentions Aphrodite by name, at line 18.[31] (A more secure mention of Aphrodite appears on the back cover, at line 18.) The front cover inscription includes mentions of the stations (*sterigmoi*) — the times when a planet stands still at the beginning or end of retrograde motion (lines 11 and 31). But there would be no need to describe these if the display included planetary pointers moving to and fro over the zodiac scale — for the stations would be immediately evident to the eye. In the same way, the mention of a conjunction (*synodon*, line 12) would not be necessary if there were moving pointers that indicated the positions of the planets, for their conjunctions with the Sun pointer would be immediately obvious. Of course, one cannot exclude the possibility that the inscription describes phenomena that the user was expected to confirm on the moving display. On the other hand, it is possible that the inscription also mentions heliacal risings and settings. Heliacal risings and settings were certainly of great interest to people contemporary with the Antikythera mechanism. But a mechanism with planet pointers moving over the zodiac would have no means of indicating them. In general, there are a large number of occurrences of the word 'days', and a substantial amount of numerical data, suggesting that one purpose of the inscription was a description of the intervals, in days, between key events of the synodic cycle. A display that focused on sequences of events in time (rather than on positions in angle) would be in keeping with the mechanism's strong emphasis on time and cycles, so prominent on the back.

Finally, if, as seems more likely than not, the display of the solar anomaly was based on System A of the Babylonian solar theory, it is worth considering whether the planetary displays might also have been rooted in Babylonian planetary theory.

Planetary Dials and a Planetary Parapegma

The front cover inscription, as it pertains to the planets, resembles the parapegma (star calendar), both in its placement (where it can be viewed simultaneously with the front display) and in its content focusing on sequences of events in time. In the case of the parapegma, key letters were inscribed at appropriate places along the zodiac scale. For example, the letters Α, Β, Γ, Δ occur in the sign of Libra and these corresponded to key letters at the beginnings of lines of writing of the parapegma inscription. Thus, when the Sun pointer reached Σ on the zodiac scale, the user could consult the parapegma and read "Σ Arcturus sets in the morning".

We propose, then, that the Venus display (for example) consisted of a small, separate dial, with a pointer that completed one revolution in approximately 584 days and that indicated the main events of Venus's synodic cycle by means of key letters. The key letters were repeated on the front cover inscription, where they were associated with short descriptions of the key events in the synodic cycle: first and last appearances, conjunctions with the Sun, onset and ending of retrograde motion, greatest elongations from the Sun. The key letters most likely appeared at the beginnings of lines of text in the front cover inscription. Unfortunately, the line beginnings and endings are all lost. Also, from top to bottom, the extant portion of the inscription covers only about a third of the available space.

According to Price, the inner radius of the zodiac dial is 62.5 mm. And the radius of the circular cap that covered the gearing for the solar eccentricity and the Moon phase display is about 30.4 mm[32] (this cap is visible in Figure 4). So there is room for planetary dials of up to about 32 mm in diameter in the interior of the zodiac scale, between the cap and the zodiac, as we suggest in Figure 13. This dial size is comparable to (and even somewhat larger than) the diameters of the subsidiary dials on the back of the mechanism. Placing the planetary phase dials here would allow them to be seen at the same time as the front cover inscription, and would make their location analogous to the locations of the small dials on the back.

The Case of Venus

We turn now to the case of Venus in particular. Early Babylonian texts, such as the goal-years texts, use for Venus the period relation: 5 synodic cycles = 8 years. However, one text also mentions that the 5/8 relation is inexact and that a correction of four days must be subtracted at the end of each cycle.[33] Similarly, in the *Almagest*, Ptolemy remarks that, for Venus, 5 synodic cycles correspond to 8 solar years less $2\frac{3}{10}$ days.[34] Other Babylonian texts, now called the ACT (Astronomical Cuneiform Texts), are based on a more accurate period relation:[35]

$$720 \text{ synodic cycles} = 1151 \text{ years.}$$

The Babylonians did not, of course arrive at this by observing Venus for 1151 years. Rather, this relation was obtained as an arithmetical modification of the simple relation, 5 synodic cycles = 8 years + correction. (Note that 1152/720 = 8/5.)

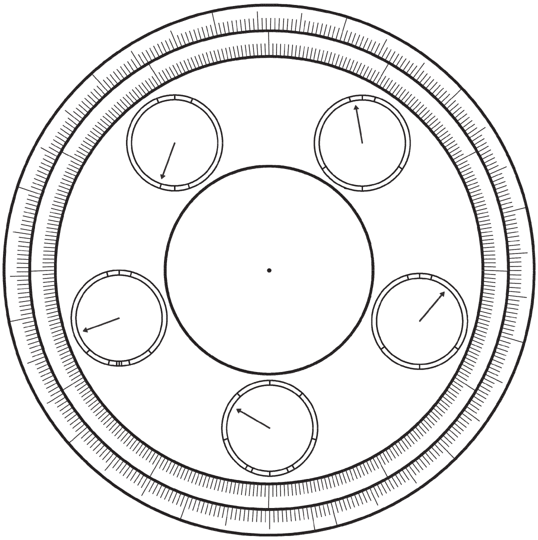

FIG. 13. Proposed reconstruction of the front face of the mechanism, showing the placement of five dials indicating planetary phases. The inner circle is the perimeter of the Moon cap.

1151 is a prime number, so the 1151/720 ratio is not expressible in terms of a product of simpler ratios. But it is also out of question to make a gear of 1151 teeth. Most attempts to incorporate a Venus display into the Antikythera mechanism have been based on the 5/8 relation. However, any display based on this proportion will be noticeably out of step with the phenomena after a single cycle. We therefore propose that the basis of the display was the more accurate ratio. The problem faced by the maker of the Antikythera mechanism, then, was to find a simple gear train that will yield the ratio 1151/720 only approximately, but with adequate precision.

The ratio to be approximated is $1151/720 \approx 1.598611$, which is extremely close to the simpler $8/5 = 1.60000$. Indeed, we could not hope to improve upon 8/5 by any two wheels with reasonably small numbers of teeth. The technique, therefore, is to step somewhat farther away from 1.6 and then to use another train of gears to come most of the way back. A reasonable way to represent 8/5 is with gears of 64 and 40 teeth, corresponding to wheel sizes in the range typical of the mechanism. Table 2 shows the ratios r given by gear combinations around 64/40.

For each possible gear ratio r, we calculate the corresponding correction factor

TABLE 2. Gear ratios around 64/40.

	62	63	64	65	66
38	1.63158				
39		1.61538	1.64103	1.66667	
40		1.57500	**1.6**	1.625	
41		1.53659	1.56098	1.58537	
42					1.57143

TABLE 3. The correction factor x for each gear ratio r.

	62	63	64	65	66
38	0.9798				
39		0.9896	0.9742	0.9592	
40		**1.0150**	0.9991	0.9838	
41		1.0404	1.0241	1.0084	
42					1.0173

x, defined by

$$r\,x = 1151/720.$$

For example, if the gear ratio $r = 65/40$ were adopted, it would be necessary to correct this by means of other gears approximating the ratio $x = (1151/720)(40/65) \approx 0.98376$. The tests of whether this is a plausible solution are (1) whether x truncates, with decent precision, after a small number of decimal places, and (2) whether this would permit x to be obtained by gears with reasonable numbers of teeth. Table 3 displays values for x.

Only for the ratio 63/40 does the value of the correction factor x truncate with good precision after a small number of decimal places. Indeed, for $r = 63/40$, we have

$$x = \frac{40}{63} \cdot \frac{1151}{720} \approx 1.014991 \approx 1.0150.$$

Adopting $x = 1.015$, then, we have

$$x = \frac{1015}{1000} = \frac{203}{200} = \frac{29}{25} \times \frac{7}{8}\,.$$

Testing the quality of the approximation, we find

$$\frac{63}{40}\,x = \frac{63}{40} \times \frac{203}{200} \approx 1.598625.$$

which is extremely close to the 1.598611 we are trying to match. None of the other starting ratios r in the table above yield an approximation approaching this quality; that is, none yields an x that can be formed by ratios of acceptably-sized integers. (But we shall see below that the case 65/39 requires special discussion.)

The quality of this approximation can be made clear in another way. Let us define n by

$$\frac{63}{40} \times \frac{203}{200} = \frac{n}{720}\,.$$

Computing, we find $n = 1151.01$, exactly, which is only one part in 10^5 from the Babylonian integer (1151) we are seeking to match. We do not mean, of course, that these gear ratios would represent the actual phenomena with an accuracy of one part in 10^5, since the Venus phenomena display some variability from one synodic cycle to the next over the 5 that approximately fill out 8 years — just that the gear ratios excellently approximate the 1151-year Babylonian period. Nor, of course, do we suppose that the maker of the Antikythera mechanism performed the arithmetical analysis in just this way; rather, he would have worked directly with fractions or perhaps with sexagesimal numbers.

TABLE 4. The correction factors x expressed sexagesimally

	62	63	64	65	66
38	0;58,47,15				
39		0;59,22,37	0;58,26,57	0;57,33	
40		**1;00,53,58**	0;59,56,52	0;59,01,32	
41		1;02,25,19	1;01,26,48	1;00,30,05	
42					1;01,02,16

We have set out the logic in decimal form for the reader's sake, but now let us see how a sexagesimal demonstration would run. Refer to Table 4, which displays the correction factors x in sexagesimal notation.[36] We seek a correction factor x which truncates with good precision after two sexagesimal places. We see again that the best candidate is 63/40: we need only add 2 parts in the third sexagesimal place to obtain

$$x = 1;00,54 = 203/200,$$

just as before.

If we seek to truncate to two sexagesimal places, the other correction factors involve larger truncation error. For 62/38, we would have to subtract 15 parts in the third place; for 63/39, we would have to add 23; for 64/41, add 12, and so on. The only close contender is 64/39, which would require addition of 3 parts in the third place.[37]

The case of 65/39 requires a word. This truncates *exactly* after two sexagesimal places. That is, $x = 0;57,33$ exactly. This might be thought an advantage; it is, however, the reverse, as it makes the method impossible to apply. With a little arithmetic, we see that 0;57,33 = 1151/1200. That is, the large prime 1151 that we were seeking to approximate has come right back in. Thus, if x truncates exactly after a small number of places, it does us no good — the whole point is to make an approximation.

Now, it is true that all of the gear trains identified for solar and lunar phenomena in the Antikythera mechanism express Babylonian period relations exactly, with no rounding. However, the 8-year period for Venus simply would not have been accurate enough for a reliable display — after only one period the display would have disagreed with the phenomena by two or three days. We have therefore inquired whether a display might be realizable by using the more accurate 1151-year Babylonian period. We find that it would be easily approximated with excellent accuracy and that such an approximation appears to call for a gear of 63 teeth. As there is, indeed, a surviving gear of 63 teeth whose function is unknown[38] (the only gear among the extant 30 that does not seem to have any role in the mechanism's solar and lunar cycle displays), we feel this proposal is now somewhat beyond the realm of mere conjecture. The 63-tooth gear could have a natural role in the Venus display, whether this was kinematic or merely a display of the phases. Because we believe the latter was the case, let us see how this gear could have been used.

The gear train can be made up in several different ways, for example

$$\frac{7}{40} \times \frac{63}{25} \times \frac{29}{8}.$$

But, as wheels of 7 or 8 teeth are rather too small, we scale these up. With some

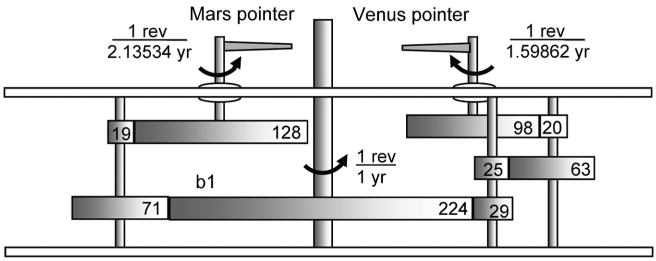

FIG. 14. Proposed mechanism for indicating the phases of Venus and Mars. For simplicity, the apparatus on the b axis for reproducing the anomalous motion of the Sun (described above and shown in Fig. 11) is not shown here.

rearranging, we obtain as a possibility:

$$\frac{98}{20} \times \frac{63}{25} \times \frac{29}{224} \quad \text{(Venus).}$$

The synodic cycle of Venus can thus be matched with excellent precision, incorporating both the unassigned 63-tooth gear and the 224-tooth drive wheel that has been previously proposed as a jumping-off point for planetary displays. A possible train is illustrated on the right side of Figure 14. In this gear train, certain features are more or less fixed, while others may still be adjusted. 29 is prime, so at least a multiple of this number must occur in the gear train. Moreover, the 63- and 224-tooth wheels actually survive, so these are not adjustable. Indeed, this reconstruction seems to resolve the ambiguity in the number of teeth on this large wheel. Freeth *et al.* made careful measurements and arrived at “223 or 224”. But now it is clear that it must be an even number. Indeed, given that it is around 220, it must be a multiple of 8. Until now, there had not appeared to be any reason for the particular tooth count of this wheel, which had seemed completely unconstrained by astronomical considerations. So this reconstruction of the Venus display provides at least a partial explanation. Of course, the denominators of the various fractions may be permuted to go with other numerators. (But, since the wheels of 224 and 63 have slightly different tooth sizes,[39] these probably did not engage directly, which provides another constraint on the possibilities.) And one could imagine wheels of 10 and 49 in place of the wheels of 20 and 98.

The Other Planets

The inscription on the front cover of the mechanism may mention Mars (Ares). While this is not completely secure, there almost certainly are references to Mercury (Hermes).[40] Therefore, it is worth asking whether there might have been displays of the phases of the other planets.

The Mars ACT are based on the period relation: 133 synodic cycles = 284 years.[41] In this case there is no need to approximate, since the ratio is factorable in terms of reasonably small integers:

$$\frac{284}{133} = \frac{4}{19} \times \frac{71}{7}$$

$$= \frac{128}{19} \times \frac{71}{224} \quad \text{(Mars).}$$

A phase display for Mars can therefore be run off the same 224-tooth gear, b1, as in the left side of Figure 14. The Mars pointer makes one revolution in one synodic period, which is about 2.13534 years according to the ACT relation. The evidence is thin here, but if this part of the reconstruction is correct, the large drive gear would be much more tightly constrained. This wheel would need to be a multiple of 8 to drive Venus and a multiple of 7 to drive Mars — thus a multiple of 56. No other plausible explanation has been offered for the tooth count of this gear.

Let us turn to the other planets. For Saturn, there is again but one ACT period relation: 256 synodic cycles = 265 years.[42] Because the number of synodic cycles in the period is divisible by 8, we may again form a train driven by the same 224-tooth gear:

$$\frac{265}{256} = \frac{105}{24} \times \frac{53}{224} \quad \text{(Saturn).}$$

For Jupiter, multiple ACT relations are attested:[43]

391 synodic cycles	= 427 years
11	12
65	71
76	83
87	95
239	261

These are not, however, all independent, since the sum or difference of two period relations will also be a period relation. Thus the periods of 71 and 95 years may be obtained by subtracting or adding the 83-year and 12-year periods. And the 261-year period is equal to the 427-year period less two 83-year periods. The scribes could, and did, thus form approximate period relations appropriate for special situations involving time intervals of longer or shorter length. All these period relations for Jupiter are not, of course, of equal accuracy. A 344-year period attested in a text published by Kugler,[44]

315 synodic cycles = 344 years,

is also not independent, as it is the difference of the periods of 427 and 83 years. This one turns out to be exactly what we need, as the number of synodic cycles is divisible by 7. And thus we can drive the synodic display for Jupiter from the same wheel of 224 teeth:

$$\frac{344}{315} = \frac{128}{45} \times \frac{86}{224} \quad \text{(Jupiter).}$$

The 344-year Jupiter period is also mentioned in a Greek papyrus of the second century A.D. that preserves a small portion of a planetary treatise, perhaps by Menelaos.[45] While this text is later than the Antikythera mechanism, it does situate the 344-year Jupiter period in a Greek context.

For Mercury, there are again multiple ACT periods attested. Neugebauer surmised that at the basis of them all was the 46-year period mentioned by a procedure text (ACT 800): 145 synodic cycles = 46 years. However, the Babylonian ephemerides use not this relation, but several others (a different relation being used for each of the four principal events in the synodic cycle, in order to satisfy certain arithmetic restrictions imposed by the speed zone system of Babylonian planetary theory):[46]

1223	synodic cycles =	388 years
1513		480
684		217
2673		848

The first and third of these, along with the 46-year period, appear to be the fundamental, independent relations. The difference between the first and third gives

539 synodic cycles = 171 years.

As 539 is divisible by 7, we may drive the synodic motion of Mercury from the wheel of 224 teeth:

$$\frac{171}{539} = \frac{96}{77} \times \frac{57}{224} \quad \text{(Mercury).}$$

Figure 15 shows in detail that the gearing can easily be accommodated in the available space. All the wheels that engage the 224-tooth wheel b1 must have gear module 0.575 mm/tooth; and, for the sake of simplicity, we have used this same module for nearly all the remaining gears as well. Only for the Mercury wheels of 77 and 96 have we used a gear module of 0.50 mm/tooth. This was not strictly necessary, but it resulted in a clearer visual separation of the trains for Mercury and the neighbouring planets. The smallest and largest of the dashed, concentric circles represent the perimeter of circular Moon cap and the inner edge of the zodiac ring, respectively. The middle dashed circle, located halfway between them, would have carried the spindles for the planetary pointers in this proposal. In the proposal of Figure 15, the pointer for Venus runs in the opposite direction to that for the other planets. This would probably not have been perceived as a serious issue, since on the back of the instrument all the subsidiary dials do not run in the same direction. However, one could easily use an idler wheel or some other device to reverse the direction of the Venus pointer, if desired.

The overall width of the Antikythera mechanism, as originally constructed, is not securely known. Freeth *et al.* in their 2006 article estimate it at 190 mm. But in the Supplementary Notes to their 2008 article, they put it at 180 mm. We have used the latter, more conservative figure. So in Figure 15, the vertical lines correspond to a separation of 180 mm. This value presumably applied to the outside dimension of

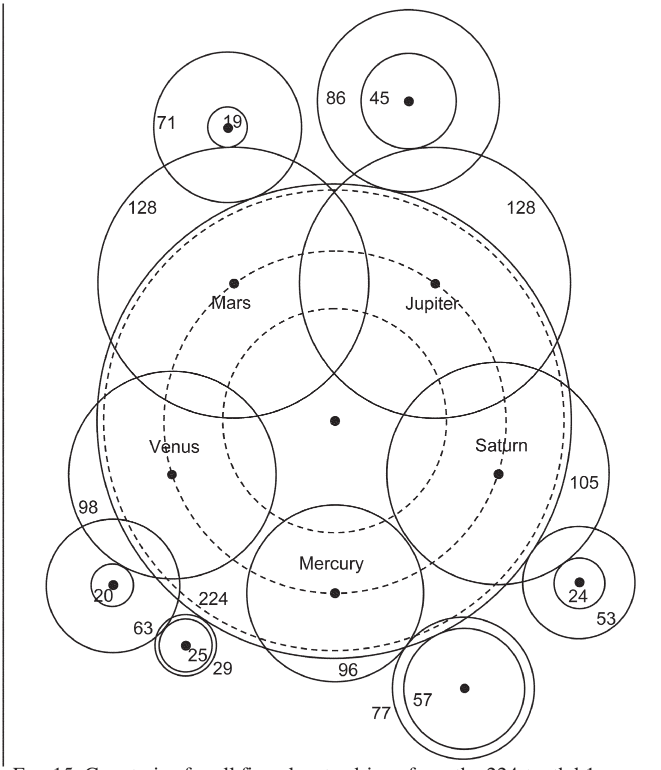

FIG. 15. Gear trains for all five planets, driven from the 224-tooth b1.

the box, so some space must be left inside to account for the thickness of the boards. Clearly, everything fits. But if it were necessary, a little more space could be obtained by adjusting the modules of the gears that do not engage the 224.

3. CONCLUSIONS AND QUERIES

In this paper, we have made a number of points, ranging from what we regard as clear demonstrations, to likely proposals, to simple possibilities. We have demonstrated that the Antikythera mechanism displayed the solar anomaly by making use of a non-uniformly divided scale, most likely the zodiac. There was therefore only a single solar pointer, which indicated not only the day of the Egyptian year, but also the true longitude of the Sun.

We cannot be certain of the details of the underlying model, which might be an arithmetical scheme based on System A of the Babylonian solar theory (which is the possibility best supported by the statistics), or an eccentrically-divided protractor (which offers advantages of simplicity and consistency). In either case, an eccentric axis of rotation for the Sun pointer was most likely a part of the design, and we have suggested a plausible mechanism for obtaining it.

We have suggested that the Antikythera mechanism may have indicated the principal events in the synodic cycles of the planets by means of subsidiary dials located inside the zodiac ring. This proposal is offered as an alternative to the more commonly suggested kinematic display of the motions of the planets according to deferent and epicycle theory. (Though it is not completely beyond the realm of possibility, we doubt that the mechanism did both.) We have shown that a phase display based on the Babylonian ACT period for Venus provides a clear role for the unassigned 63-tooth wheel. Moreover, we have shown that the 224-tooth gear b1 could drive the subsidiary dials for all five planets, with gear trains based on a coherent body of period relations derived, ultimately, from Babylonian sources. This construction also provides the only explanation offered to date for the 224-tooth count of b1: it needed to be a multiple of 8 to drive Venus and Saturn, and a multiple of 7 to drive Mars, Jupiter and Mercury — hence a multiple of 56. While our planetary proposal is necessarily far more speculative than the solar section of the paper, we have also shown that the necessary dials, as well as the gearing, can be accommodated in the space available.

Gear b1 evidently needed to be large. A tooth count of 168 (= 3 × 56) would have met all the constraints imposed by the planetary periods, and it is not hard to show that gear trains based on such a count could have been incorporated into the mechanism. The maker opted for 224. And even with the tooth count at 224, b1 evidently was still not quite large enough, for as we have seen, the maker oversized the teeth of this wheel by about 16%. Why? The chief objection one might raise against our proposal is that we have so far given no explanation of the small, round and rectangular holes on the four spokes of b1. Unless more material is discovered, it is unlikely that we will ever know just what these were for. Wright, as well as Freeth *et al.*, have proposed that these holes on the spokes were the mounting places for complexes of epicyclic gearing that produced the motions of the planets. But we suggest that the field of possibilities should be broadened to include non-technical displays. These holes might, for example, have been the mounting places for small statues or figurines, indicative of the four seasons, or perhaps of the winds. As b1 turned in the course of a year, one after the other of these could have appeared in a small window on the front face, between the zodiac and the circular cap. Boreas, the north wind, might appear in the window around the winter solstice; Zephyros, the west wind, in the spring; and the Etesian winds might appear in late summer — to name but a few wind-season associations that are common in the ancient writers. The association of particular winds with particular times of year is also common (though not universal) in the parapegmata. And it is noteworthy that winds do not appear in the parapegma of the Antikythera mechanism,[47] so a visual display would not duplicate the inscription. A mobile wind (or season) display in the Antikythera mechanism would have a strong affiliation with medieval astronomical clocks, which sometimes included moving three-dimensional images. It would also resonate with the ancient interest in *thaumatopoiïkē*, the "wonder-working" (art), practised and described by Heron of Alexandria,[48] but also ranged by Geminos among the branches

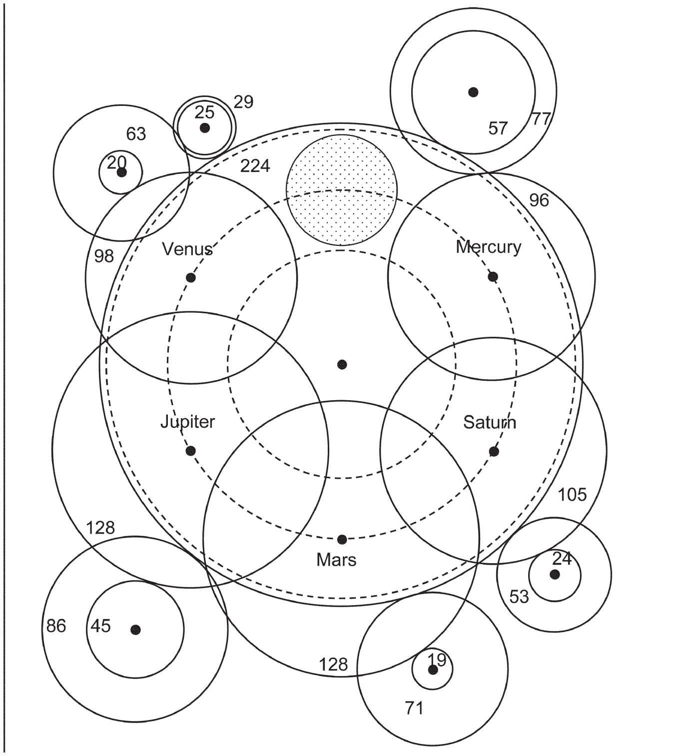

FIG. 16. Gear trains for all five planets, incorporating space for a window to display wind or season indicators.

of mechanics, along with *sphairopoiïa*.[49] Finally, it would be consistent with the large array of iconographical evidence showing wind and season symbolism in conjunction with the zodiac.[50] Figure 16 shows that there was indeed room for such a display in the mechanism. Perhaps the spaces between the spokes of b1 played some role in a mechanism that allowed the wind god (or seasonal symbol) to move out through the window (like the cuckoo on a modern clock). In the absence of material evidence, this remains no more than a possibility. But it should serve to remind us that an ancient mechanic may have seen more possible uses for those holes on the spokes of b1, as well as the spaces between the spokes, than we might.

We conclude by making some observations on the astronomical sources of the Antikythera mechanism. All the numerical relations behind the displays were known to the Babylonians. But because of the mechanism's likely date (second century to early first century B.C.), it is natural to ask whether it might reflect the astronomical theories of Hipparchos. We should perhaps be careful about focusing on this one figure, as important as he is, for we know so little about Greek astronomy in the second century B.C. that we cannot be sure what other theories may have been circulating at the time. So far, the only elements of Antikythera astronomy that might plausibly reflect Hipparchos's direct influence are the epicyclic gearing for the lunar anomaly, described by Freeth *et al.*, and the device for handling the solar anomaly, described here (the gearing for the eccentric axis, as well as the nonuniform division of the zodiac, if it is indeed based on an eccentric circle rather than on System A).

Freeth *et al.* have shown that the epicyclic gearing (with its pin-and-slot mechanism) riding on gear e3 yields a variable motion geometrically equivalent to that of Hipparchos's epicyclic lunar theory. Thus, in parts of their paper, Freeth *et al.* characterize the lunar epicyclic gearing as a mechanical realization of Hipparchos's theory. But they also note that the mechanism gives a somewhat larger amplitude for the lunar anomaly than one would expect from Hipparchos's theory, and that the Babylonian values of the anomaly generally are also somewhat larger.[51] Thus they implicitly leave open the possibility that the ancient mechanic was modelling, not Hipparchos's geometrical theory, but the Babylonians' arithmetical theory of the Moon, in the same way that he modelled the more abstract exeligmos. Of course, the difference in amplitude might simply be due to imperfections in the construction. More importantly, as Freeth *et al.* acknowledge,[52] the lunar periods in their reconstruction are not those that Ptolemy attributes to Hipparchos. For example, the lunar mechanism is based in part on the Babylonian 'saros' relation, which requires that 223 synodic months = 239 anomalistic months. But Ptolemy says that Hipparchos found that 4267 synodic months = 4573 anomalistic months.[53] Unknown to Ptolemy, this was not original with Hipparchos, as it is equivalent to another relation that turns up in System B of the Babylonian lunar theory: 251 synodic months = 269 anomalistic months.[54] Now these ratios are not far apart, but if the maker of the Antikythera mechanism were drawing directly on Hipparchos, one is left to wonder why he used a Hipparchian epicycle, but a non-Hipparchian period relation. Freeth *et al.* speculate that the attraction was the ability to drive the lunar display from the same saros train that animates the eclipse display on the back of the instrument. In any case, the evidence for the direct influence of Hipparchos is somewhat mixed.

Similarly, while it is possible that the nonuniform division of the zodiac was based on an eccentric circle theory, the implied numerical value of the eccentricity does not agree very well with Hipparchos's value. Whether this was the result of accident or design, we cannot say. More importantly, we cannot exclude the possibility that the nonuniform division of the zodiac scale on the Antikythera mechanism was actually based on the Babylonian solar theory of System A.

The ACT relations for Venus and Mars that we find connected with the 63- and 224-tooth gears are not the ones reported by Ptolemy in the *Almagest*,[55] and so we have no evidence connecting them to Hipparchos. It is noteworthy that the Mars and Venus period relations that Ptolemy attributes to Hipparchos (though they, too, were of Babylonian origin) do not lead to an explanation for the 63- and 224-tooth gears. So, while the Antikythera mechanism does include features that could plausibly owe something to Hipparchian astronomy, it also includes features independent of Hipparchos. And certainly, all the numerical relations are pre-Hipparchian. The emphasis upon planetary phases is quite consistent with Babylonian influence, as well as with the state of Greek astronomy in the second century B.C. After all, as Ptolemy tells us, Hipparchos had not attempted a theory of the planets.[56] Though deferent-and-epicycle planetary theories existed by his time, they were not yet accurate enough to be useful for numerical computation.

If, as we have proposed, the planetary displays focused on synodic phenomena and if, as the statistics suggest, the display of the solar anomaly on the zodiac scale was based on System A, then it is possible that the Antikythera mechanism reflects a slightly earlier stage of Greek astronomy, in which geometrical models and Babylonian arithmetical methods were still in the beginning stages of the process of integration. Some might find it a disappointment if the Antikythera mechanism does not reflect a purely Greek cosmos of deferent-and-epicycle theory. But, actually, in such a case the mechanism would be even more important, as it would have more to teach us about the astronomy of a period for which Greek sources are so few. Finally, if the maker of the Antikythera mechanism used gears to model Babylonian astronomical cycles, and if, as is likely, the mechanism reflects a craft tradition going back to the time of Archimedes, this raises the fascinating, but unprovable, possibility that epicycles and deferents entered Greek astronomy, not because of natural philosophical considerations, but because some geometer applied a geometrical image of gearing to a cosmic problem.[57]

ACKNOWLEDGEMENTS

We are very grateful to the Antikythera Mechanism Research Project for preparing the x-ray slices of Fragment C that we used in this study and for permission (via Professor Mike Edmunds) to reproduce the images here. Thanks are due to the National Archaeological Museum in Athens. X-Tek Systems Ltd was instrumental in the original collection of the x-ray images, as was Hewlett-Packard Inc for the surface image. Marcelo Di Cocco provided advice about Photoshop, and James Bernhard helped with statistical analysis. Agamemnon Tselikas and Yanis Bitsakis generously shared their new text of the front cover inscription in advance of publication. Dennis Duke, Alexander Jones, and Michael Wright read an early draft of this paper and made many suggestions that helped us improve the paper; but they do not, of course, necessarily agree with the views we express here. A subvention from the Physics Department of the University of Puget Sound permitted this *Journal* to enlarge the February issue so that the entire article could appear in a single number. The authors would also like to express their gratitude for the Fulbright grant that permitted Christián Carman to work for a semester with James Evans and Alan Thorndike at the University of Puget Sound.

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2. Wright, "A planetarium display" (ref. 1).
3. Freeth *et al.*, "Decoding" (ref. 1).
4. "Supplementary information" to Freeth *et al.*, "Decoding" (ref. 1), pp. 8 and 10–12.
5. "Supplementary information" to Freeth *et al.*, "Calendars" (ref. 1), p. 5.
6. Wright, "A planetarium display" (ref. 1). A film of Wright's reconstruction in motion can be seen at http://www.youtube.com/watch?v=ZrfMFhrgOFc. This film was made in 2008 by the science writer Jo Marchant, who was then working for *New scientist*.
7. Freeth *et al.*, "Calendars" (ref. 1).
8. The possibility of an Olympiad display was overlooked by all the principal investigators, but it was actually suggested by Victor J. Kean, in a popular book, *The ancient Greek computer from Rhodes known as the Antikythera mechanism* (Anixi, 1995), 77–84. Our thanks to Alexander Jones for this reference.
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11. For example, see Roderick and Marjorie Webster, *Western astrolabes* (Adler Planetarium and Astronomy Museum, Chicago, 1998), no. 1 (an English astrolabe of around A.D. 1250; and nos. 2 and 3 (astrolabes probably made by Johanne Fusoris at Paris around 1400).
12. See the Hartmann astrolabes nos. 5 and 6 (from 1532 and 1540) in Webster and Webster, *Western astrolabes* (ref. 11).
13. Price, *Gears from the Greeks* (ref. 1), 18.
14. The zodiac scale of the Antikythera mechanism reflects the common Greek nomenclature of its day

by calling this zodiac sign ΧΗΛΑΙ, Chelai, the "Claws" (of the Scorpion). Price, *Gears from the Greeks* (ref. 1), 17–18, mistakenly read ΧΥΛΑΙ, an error that is still sometimes repeated. Our thanks to Michael Wright for pointing this out.

15. Price's discussion, in *Gears from the Greeks* (ref. 1), 18, contains an arithmetical or other error that probably prevented him from recognizing that, for the interval on the rings that he analysed, the number of degrees exceeds the number of days, which would be impossible on uniformly divided, concentric rings. Thus he missed concluding that one of the two rings was deliberately divided nonuniformly.

16. The holes underneath the Egyptian calendar scale, which are invisible to surface examination, were described by Price, *Gears from the Greeks* (ref. 1), 18, but were first explained by Wright, "In the steps of the master mechanic" (ref. 1), 4. The number of holes that existed on the original mechanism has been variously estimated as 360 by Price, 365 by Wright, and falling in the range 363–365 by Freeth *et al.*, in "Decoding" (ref. 1, caption to Fig. 2). In principle, these holes should provide another method of estimating the solar anomaly intended by the designer of the Antikythera mechanism. But because of the errors in the spacing the holes, and the resulting uncertainty in their number, the question is a delicate one. We shall deal with it in a separate paper.

17. We reckon longitudes from the spring equinoctial point. We treat the long mark at the beginning of a zodiac sign as representing 0º of that sign, although we recognize that it is possible that an ancient user might have understood this as the first degree.

18. In the case of longitude 234, which falls between day marks 68 and 69, since day mark 69 cannot be seen, we extrapolated a value for 234 from day marks 67 and 68. This introduces negligible error, as longitude 234 lies only a very small distance beyond the day mark 68.

19. James Evans, *The history and practice of ancient astronomy* (New York, 1998), 234. Note that in this expression, q is given as a function of the true anomaly, $\lambda - A$. Usually (as when calculating the longitude of the Sun for a given date) one wants q as a function of the mean anomaly, which involves a more complicated expression.

20. The best-fit eccentricity may be expressed in Ptolemy's terms as 2.142 parts out of 60, compared with the Hipparchos value of 2.5.

21. A similar argument applies to the holes drilled into the plate beneath the calendar scale: the holes cannot have been intended to be unequally spaced, as this would result in a non-uniform shift of the Egyptian calendar with respect to the zodiac signs.

22. Vitruvius explicitly informs his readers that the zodiac is divided nonuniformly on the anaphoric clock (*On architecture*, ix, 8.8).

23. Statistical methods confirm the common-sense impression from Fig. 9 that the best-fit eccentric model cannot be ruled out. First, we may estimate the standard deviation σ of the data distribution in Fig. 9 by temporarily assuming that System A is the underlying true theory. With $N = 66$ points, and the sum of squared residuals mentioned, this gives $\sigma = 0.16°$. We assume that this characterizes the original placement of the divisions as well as our measuring process. Now, we examine the possibility that the data are actually described by the best-fit eccentric. Then, for the eccentric model, χ^2 = sum of squared residuals/σ^2 = 103. The probability Q that χ^2 could be this large or larger simply by chance (assuming the validity of the best-fit eccentric) then turns out to be around 0.002 — a possibility that cannot be rejected. In model testing, "it is not uncommon to deem acceptable on equal terms any models with, say, $Q > 0.001$". See William H. Press *et al.*, *Numerical recipes: The art of scientific computing* (Cambridge, 1986), 503. By contrast, the Hipparchos value of $e = 0.0417$ has a sum of squared residuals of 5.09 deg^2, which gives $\chi^2 = 199$ and a probability of 3×10^{-15}. This, of course, means only that the Hipparchos value does not describe the actual data. But it does not prove that the mechanician was not aiming at the Hipparchos value. There could be, for example, a centre-placement error or some other systematic error.

24. Price, *Gears from the Greeks* (ref. 1), 16.
25. "Supplementary information" to Freeth *et al.*, "Calendars" (ref. 1), p. 24.
26. Wright, "Early history of the moon phase display" (ref. 1).
27. Or perhaps the maker believed that the Moon's motion contains a zodiacal anomaly identical to the Sun's. We do not know of any examples of such a theory, so we do not seriously entertain this possibility.
28. Musée du Louvre, Département des Antiquités Egyptiennes, inv. n. 2325. (This is the papyrus sometimes called "The Art of Eudoxus".) Text: F. Blass, *Eudoxi ars astronomica* (Kiel, 1887). There is a French translation by Paul Tannery, *Recherches sur l'histoire de l'astronomie ancienne* (Paris, 1893), 283–300.
29. Freeth *et al.*, "Decoding" (ref. 1).
30. In our proposal, the solar input to the Moon phase display is based on the Sun's mean (rather than true) motion. But the solar anomaly only affects the times of new and full moons by about 4 hours — a quantity that would be imperceptible on the Moon phase display. (The Sun's maximum equation is about 2º. At the Moon's rate of motion of about 12º per day, it would take 1/6 day of motion to compensate for a 2º offset.)
31. The line numbers are from the preliminary text of the front cover inscription published in the "Supplementary information" to Freeth *et al.*, "Decoding" (ref. 1), pp. 8 and 10–12. Agamemnon Tselikas and Yanis Bitsakis are in the course of preparing a new version of the front cover inscription, based on x-ray CT, and they presented a report on their progress in "The front cover plate of the Antikythera mechanism" (oral presentation), XXIII International Congress of History of Science and Technology, Budapest, 2009. Substantial changes in the text have been made (and the lines numbers changed by 1), so the preliminary text should be used with caution until the revised text is published. For this reason, we refrain from detailed discussion of the inscription.
32. Price, *Gears from the Greeks* (ref. 1), 20.
33. For an English translation of this text (British Museum, Shemtomb 135), see B. L. van der Waerden, *Science awakening*, ii: *The birth of astronomy* (New York, 1974), 107–8.
34. Ptolemy, *Almagest* ix, 3. G. J. Toomer, *Ptolemy's Almagest* (London, 1984), 424.
35. O. Neugebauer, *Astronomical cuneiform texts* (London, 1955), ii, 300.
36. We use the standard notation in which the integer and fractional parts of the number are separated by a semicolon, and the successive sexagesimal places are separated by commas.
37. But this (for $r = 64/39$) would result in $x = 0;58,27 = 7 \times 167/1200$. A prime 167 gear would be cumbersome, though of course workable. But the truncation error is worse than for 63/40, so this option offers no advantage at all.
38. Freeth *et al.*, "Decoding" (ref. 1), 589–90.
39. "Supplementary information" to Freeth *et al.*, "Decoding" (ref. 1), p. 15.
40. Tselikas and Bitsakis, "The front cover plate of the Antikythera mechanism" (oral presentation, ref. 31).
41. Neugebauer, *Astronomical cuneiform texts* (ref. 35), ii, 302.
42. Neugebauer, *Astronomical cuneiform texts* (ref. 35), ii, 313.
43. Neugebauer, *Astronomical cuneiform texts* (ref. 35), ii, 307.
44. British Museum, Sp. II, 985, published in F. X. Kugler, *Sternkunde und Sterndienst in Babel*, i (Münster, 1913), 49. Cited in Neugebauer, *Astronomical cuneiform texts* (ref. 35), ii, 307, n. 17.
45. *P. Oxy.* 4133, in Alexander Jones, *Astronomical papyri from Oxyrhynchus* (*Memoirs of the American Philosophical Society*, ccxxxiii (2 vols. bound as one, Philadelphia, 1999)), ii, 2–5, with discussion

in vol. i, 69–80.

46. Neugebauer, *Astronomical cuneiform texts* (ref. 35), ii, 287–8.

47. No winds are mentioned in the fragmentary parapegma text published by Price. Alexander Jones is working on a new edition of the parapegma inscription, and has kindly confirmed that there is "not the slightest trace of weather" (personal correspondence).

48. Joseph Gouge Greenwood and Bennet Woodcroft (transl.), *The pneumatics of Hero of Alexandria* (London, 1851; reprinted London and New York, 1971).

49. James Evans and J. Lennart Berggren, *Geminos's Introduction to the Phenomena: A translation and study of a Hellenistic survey of astronomy* (Princeton, 2006), 43–48 and 246–9. *Sphairopoïïa* is "sphere-making", the branch of mechanics devoted to constructing images of the heavens, such as the Antikythera mechanism.

50. For example, the ivory astrological tablets of Grand (2nd century A.D.) have a central zodiac, with four winds in the corners; and this is true as well of the marble Bianchini Tablet: J.-H. Abry (ed.), *Les tablettes astrologiques de Grand (Vosges)* (Lyon, 1993). A Mithraic relief from Sidon (probably second century A.D.), now in the Louvre, has a prominent zodiac surrounding the tauroctony, with busts representing the four seasons in the corners: Cécile Giroire and Daniel Roger, *Roman art from the Louvre* (New York, 2007), 243–5. Examples could readily be multiplied.

51. Freeth *et al.*, "Decoding" (ref. 1), 590, caption to Fig. 6.

52. "Supplementary information" to Freeth *et al.*, "Decoding" (ref. 1), p. 27.

53. Ptolemy, *Almagest* iv, 2.

54. See the discussion in Otto Neugebauer, *A history of ancient mathematical astronomy* (Berlin, 1975), 309–12.

55. For Venus, Ptolemy (*Almagest* ix, 2) uses a modification of 5 synodic cycles = 8 years, and for Mars, a modification of 37 synodic cycles = 79 years.

56. Ptolemy, *Almagest* ix, 2.

57. Note added while the article was in proof: The Antikythera Mechanism Research Project has recently estimated, by fresh study of the x-rays, that the interior width of the box housing the Antikythera mechanism was 164 mm. The reconstructions of Figs 15 and 16 will both fit inside such a width. The interior height is estimated at 314 mm, and the exterior dimensions at 190 × 340 mm. Our thanks to Tony Freeth for this information.